STUDY OF ECONOMICS APPLICATIONS IN CERTAIN NON-LINEAR PROGRAMMING DUALITY PROBLEMS

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By

Harshita Gupta
(Roll No. 2K22/MSCMAT/16)

Anuja
(Roll No. 2K22/MSCMAT/55)

Under the Supervision of
Dr. L.N. Das
Professor, Department of Applied Mathematics
Delhi Technological University

To the
Department of Applied Mathematics

Delhi Technological University
(Formerly Delhi College of Engineering)
Shahbad Daulatpur, Main Bawana Road Delhi 110042, India

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Harshita Gupta

Anuja

Delhi Technological University
(Formerly Delhi College of Engineering)
Shahbad Daulatpur, Main Bawana Road Delhi 110042

CANDIDATE'S DECLARATION

We Harshita Gupta and Anuja hereby certify that the work which is being presented in the thesis entitled Study of Economics applications in certain non-linear programming duality problems in partial fulfillment of the requirements for the award of the Degree of Master in science, submitted in the Department of Applied Mathematics, Delhi Technological University is an authentic record of my own work carried out during the period from 2023 to 2024 under the supervision of Dr. L.N Das.

The matter presented in the thesis has not been submitted by me for the award of any other degree of this or any other institute.

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This is to certify that the student has incorporated all the corrections suggested by the examiners in the thesis and the statement made by the candidate is correct to the best of our knowledge.

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Delhi Technological University

(Formerly Delhi College of Engineering)

Shahbad Daulatpur, Main Bawana Road Delhi 110042

CERTIFICATE

Certified that **Harshita Gupta** (2K22/MSCMAT/16) and **Anuja** (2K22/MSCMAT/55) has carried out their search work presented in this thesis entitled "**Study of Economics applications in certain non-linear programming duality problems"** for the award of **Master of Science** from Department of Applied Mathematics, Delhi Technological University, Delhi, under my supervision. The thesis embodies results of original work, and studies are carried out by the students herself and the contents of the thesis do not form the basis for the award of any other degree to the candidate or to anybody else from this or any other University.

Signature

Dr. L.N Das

Professor, Department of Applied Mathematics

Delhi Technological University

Place: Delhi

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Harshita Gupta , Anuja

ABSTRACT

The duality of nonlinear programming problems can provide interesting economic interpretations of nonlinear programming resource allocation models. An economic interpretation of dual problems specifies how changing the value of resources can change the optimal solution. In the first parts of this article, we will briefly introduce the concept of linear and non-linear programming. This article also provides some effective methods for solving nonlinear programming problems such as Lagrangian Method, Karush-Kuhn Tucker (KKT) conditions along with examples. Subsequently, the concept of duality for nonlinear programming is introduced. Various types of duals like Wolfe dual, Lagrangian dual are discussed in detail along with their examples. The Economic interpretation of duality is then explained in detail. An example of economic interpretation based on real life is also discussed.

Keywords: Duality, KKT method, Lagrangian method, Economic interpretation, Wolfe Dual, Lagrangian Dual

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LIST OF SYMBOLS

1. λ - Lambda
2. ∑ - Sigma
3. ∂ - Delta
4. ∇ - Nabla
5. $\boldsymbol{\mu}$ - Mu
6. ∈ - belongs to
7. ⊆ - subset
8. ∀ - for all
9. ∑ - N-Ary Summation
10. ∪ - union
11. ω - omega
12. π – pi
13. ρ - Rho
14. Δ - Delta

LIST OF ABBREVIATIONS

KKT	Karush – Kuhn Tucker
NLPP	Non – Linear Programming Problem
LPP	Linear Programming Problem
OF	Objective Function
CP	Canonical Primal
CD	Canonical Dual
CPP	Convex Programming Problem
$\mathbf{H^B}$	Bordered Hessian Matrix
POP	Portfolio Optimization Problem
ALM	Asset Liability Management
OOPS	Object- Oriented Parallel Solver
QP	Quadratic Programming
SQP	Sequential Quadratic Programming

CHAPTER 1

NON-LINEAR PROGRAMMING PROBLEM: OVERVIEW

1.1 Convex and Concave functions

In creating models, concave and convex functions are essential.
Convex set: Consider S be a non-empty set and assume arbitrary points x_1 and x_2 of S , then S be convex set if for any $\lambda \in (0,1)$, $\lambda x_1 + (1-\lambda)x_2 \in S$.

Convex function: Consider $S \subseteq R^n$ to be convex set and $f : S \rightarrow R$, f(x) be convex when

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y) , \lambda \in [0,1] , x,y \in S$$

Concave function: Let $S \subseteq R^n$ be convex set and $f : S \rightarrow R$, f(x) is concave if,

$$f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda) f(y) , \lambda \in [0,1] , x,y \in S$$

1.2 Non Linear Programming Problem (NLPP)

A Linear Programming Problem (LPP) is a special case of NLPP in which there are specific linear restrictions and a linear objective function (OF) that needs to be minimized or maximized.
Standard form of LPP is:

$$\text{Max } z = p_1x_1 + p_2x_2 + \ldots + p_nx_n$$

st

$$q_{11}x_1 + q_{12}x_2 + \ldots + q_{1n}x_n = b_1$$

$$q_{21}x_1 + q_{22}x_2 + \ldots + q_{2n}x_n = b_2$$

$$\ldots$$

$$q_{m1}x_1 + q_{m2}x_2 + \ldots + q_{mn}x_n = b_m$$

$$xi \geq 0 \text{ , } \forall\, i = 1 \text{ to } m$$

where $p_i \in \mathbb{R}^{nx1}$, $q_i \in \mathbb{R}^{mxn}$, $b_i \in \mathbb{R}^{mx1}$ are real numbers. Variables x_i , i=1 to n are distinct

In compact vector notation, the standard problem becomes:

$$\text{Max}\{P\}^T \text{ as } Z\{X\}$$

st

$$Q\{X\} = \{B\}$$

$$\{X\} \geq 0$$

Real-world optimization problems that can be formulated using linear objective functions as well as linear equality or inequality constraints are called linear optimization, while if such a problem cannot be formulated, this optimization problem is nonlinear. The accuracy rate of nonlinear programming is higher than that of linear programming and it can be applied to large-scale systems. In NLPP either we have nonlinear objective function or the constraints defining the feasible region are nonlinear.

The general form of NLPP is:-[1]

$$\text{Max } f(x_1, x_2, \dots, x_n),$$

st

$$g_1(x_1, x_2, \dots, x_n) \leq b_1 ,$$

$$\dots$$

$$g_m(x_1, x_2, \dots, x_n) \leq b_m$$

where g_1 to g_m, the constraint functions, are given and,

$f(x_1, x_2, \dots, x_n) = \sum c_j x_j$, (i=1 to m)

and $g_i(x_1, x_2, \dots, x_n) = \sum a_{ij} x_j$ (j =1 to n).

In a compact vector notation, the standard problem becomes:[2]

$$\text{Max } f(x) ,$$

st

$g_i(x) \leq b_i$ (i = 1 to m).

For a linear programming, a feasible convex region is one that is described by a less than inequality with a convex function. For an NLPP i.e. $g_i(x)$ for i = 1 to m are convex function and points x = y and x = z satisfies $g_i(x) \leq b_i$, i = 1 to m , $\lambda \in [0,1]$,
$\lambda y + (1-\lambda) z$ is also feasible as the inequality
$g_i(\lambda y + (1-\lambda)z) \leq \lambda g_i(y) + (1-\lambda) g_i(z) \leq \lambda b_i + (1-\lambda) b_i = b_i$
holds for every constraint. When the constraints are given by ≥ inequality and functions are convex, then feasible region is convex.

Ques: Product of one unit of bottle by company H required 1 hr of production time and A requires 2 hr of production time. Having Total available time 100 hrs . Two firms H and A manufacture same product. Sale price relation for two company is given by the following table.

Company	Selling Price	Cost Function	Required quantity	Price/unit
H	20	$400x_1+0.2x_1^2 + 20$	$200-8p_1$	p_1
A	10	$200x_2+0.5x_2^2 + 30$	$800-2p_2$	p_2

Both company requires electricity where company H requires 8 units/ unit production and A requires 10 units/ unit production having supply limits to 400 units per day. Formulate the NLPP.

Solution: Let the company H and A produces x_1 and x_2 units respectively. Cost function is given by:

$C_1=400x_1+0.2x_1^2 + 20$ for H
$C_2=200x_2+0.5x_2^2+30$ for A
and, $x_1 = 200-8p_1$, $x_2 = 800-2p_2$
$p_1 = \frac{200-x1}{8}$, $p_2 = \frac{800-x2}{2}$
$p_1 = 25 - 0.125 x_1$ and $p_2 = 400 - 0.5x_2$
Total revenue = $p_1x_1 + p_2x_2$
$R = (25 - 0.125 x_1)x_1 + (400 - 0.5x_2)x_2$
$= 25x_1 - 0.125x_1^2 + 400x_2 - 0.5x_2^2$

Therefore, Profit $z = R - C_1 - C_2$

$= (25x_1 - 0.125x_1^2 + 400x_2 - 0.5x^{2}) – (400x_1 + 0.2x^{2} + 20) - (200x_2 + 0.5x_2^2 + 30)$

$= 25x_1 - 0.125x_1^2 + 400x_2 - 0.5x_2^2 - 400x_1 - 0.2x_1^2 - 200x_2 - 0.5x^{2} - 30$

$= -375x_1 - 0.325x_1^2 + 200x_2 - x_2^2 – 50$

Therefore , NLPP becomes :

Max $z = -375x_1 - 0.325x_1^2 + 200x_2 - x_2^2 - 50$

Subject to

$8x_1 + 10x_2 \leq 400$

$x_1 + 2x_2 \leq 100$

$x_1 , x_2 \geq 0$

NLPP has various types and no single algorithm can solve every kind of NLPP. Various techniques have been created to address various NLPP types.

1.2.1 Various types of NLPP are [3]

1. Unconstrained optimization:

An optimization problem without any constraint is known as unconstraint optimization problem. Standard form is:

Maximize $f(x)$ for all $x = (x_1,x_2, \dots ,x_n)$

When f(x) is a differentiable function, the prerequisite for $x = x^*$ to be the optimal solution be:

$$\partial f / \partial x_j (x = x^*) = 0 \quad , j = 1 , 2, \dots , n$$

For concave function f, the aforementioned requirement is also satisfied.

The previous sufficient and necessary conditions marginally alter and become when the variable x_j lacks non-negativity constraints, $x_j \geq 0$.

$$\partial f/\partial x_i = \begin{cases} \leq 0 , & at\ x = x* \ if \ xj* = 0 \\ = 0, & at\ x = x* \ if \ xj* > 0 \end{cases} \quad \forall j$$

2. Linearly constrained optimization problem:

In these problems, we consider the feasible region of LPP along with one nonlinear function, and the simplex method can be extended to handle such issues. These problems have constraints that fully satisfy the linear programming. In this case, all $g_i(x)$ are linear however, f(x), the OF, is not linear.

3. Quadratic programming problem:

For these situations, linear constraints and a quadratic objective function (f(x)) are needed.

4. Convex programming problem (CPP) :
Assuming maximizing the OF with
a. Function f(x) exhibits convexity.
b. Each $g_i(x)$ have convexity.
In this case the local maxima become the global maxima. In case of minimization, f is concave function.

5. Separable Programming Problems:
If f(x) is separable, then functions of individual variables can be separated into sums of their functions. Separable functions are those whose terms involve only one variable.
$f(x) = \sum_{j=1}^{n} f_j(x_j)$ where terms involving x_j are included in each $f_j(x_j)$. An extension of convex programming, separable programming requires that f(x) and $g_i(x)$ be separable functions.

6. Non-Convex Programming Problem:
NLPP not satisfying the condition of convex programming problem are said to be non-convex programming problems. An algorithm that finds the best answer to these kinds of problems does not exist.

7. Geometric Programming Problem:
When NLPP is used to solve economic and statistical problems, the constraints and OF become:

$$g(x) = \sum_{i=1}^{N} c_i P_i(x)$$

where, $P_i(x) = x_1^{a_{i1}} x_2^{a_{i2}} \ldots x_n^{a_{in}}$ i= 1 to N , and c_i and a_{ij} represents physical constants, x_j is the design variable. These functions are neither convex nor concave

8. Fractional Programming Problems:
The OF in these kinds of problems have the form :

$$\text{Max } f(x) = \frac{f_1(x)}{f_2(x)}$$

Best way to solve the fractional programming problem is by transforming it into standard problem for which solution is available.

1.2.2 We now discuss some methods of solving NLPP:

a) Lagrangian Multiplier Method:
This method is pertinent for solving NLPP having constraints with equality sign.
Consider general NLPP with equality constraints,

$$\text{Max/Min } z = f(X)$$

st

$$g_i(X) = 0 \quad , \quad i = 1 \text{ to } m$$

$$X \geq 0 \quad , \quad X = (x_1 , x_2, \dots ,x_n)$$

To ascertain the prerequisite for the minimum or maximum value of z, we create new function L by adding a multiplier λ , also referred to as the Lagrange multiplier or Lagrangian.

$$L(X, \lambda) = f(X) + \lambda\, g(X) \qquad \lambda \text{ is unrestricted in sign}$$

The given constrained optimization problem is transformed into an unconstrained optimization problem by using Lagrangian multiplier. Necessary conditions are given by:

$$\frac{\partial L}{\partial X} = 0 \ , \quad \frac{\partial L}{\partial \lambda} = 0$$

Solving these two equations we get the stationary points (X^*,λ^*). Solve the above equations to find $x_1 , x_2 , x_3,\dots , x_n , \lambda$. Sufficient condition for general NLPP at the stationary points (X^*,λ^*) can be found by two methods:

Method 1

Necessary condition becomes sufficient condition for max(min) if :

1. f(x), the objective function, is concave (convex).
2. Constraints have equality sign

Method 2

In this method we define a bordered Hessian Matrix H^B as $H^B = \begin{bmatrix} 0 & U \\ U^T & V \end{bmatrix}$

H^B is a (m + n) x (m + n) matrix and m being composed of m constraints and n variables.

$$U = \left[\frac{\partial g}{\partial X}\right] = \left[\frac{\partial g}{\partial x_1}\frac{\partial g}{\partial x_2}\cdots\frac{\partial g}{\partial x_n}\right]$$

$$V = \left[\frac{\partial^2 L}{\partial X^2}\right] = \begin{bmatrix} \frac{\partial^2 L}{\partial x_1^2} & \cdots & \frac{\partial^2 L}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 L}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 L}{\partial x_n^2} \end{bmatrix}$$

Now at the stationary point (X^*, λ^*). Calculate the final principal minors (n-m) of Hessian matrix H^B beginning with principle minor of order 2m+1. The stationary points is a maximum point if the principle minors forms Alternative signs starting with $(-1)^{m+n}$ and if the principle minors exhibit signs of the form $(-1)^m$, the stationary point is the minimum point.

b) Karush-Kuhn Tucker Condition (KKT Conditions):
KKT condition is used for finding solution of NLPP having constraints with inequality. It is also called first derivative test. Consider general NLPP with inequality constraints,

$$\text{Max/Min } z = f(x)$$

st

$$g_i(x) \le b_i, \quad i = (1 \text{ to } m)$$

$$x \ge 0, \quad x = (x_1, x_2, \ldots, x_n)$$

General NLPP for Maximization case,

$$\text{Max } z = f(x)$$

st ... (1.1)

$$g_i(x) \le b_i$$

Convert each inequality constraint into equality constraint by adding the non negative slack variables s_i^2. Consequently, the initial problem turns into

$$\text{Max } f(x)$$

st

$$g_i(x) + s_i^2 = b \quad x \ge 0 \quad \ldots (1.2)$$

$$\text{Max } f(x)$$

St ... (1.3)

$$h_i(x) = 0 \quad \text{where} \quad h_i(x) = g_i(x) + s_i^2 - b_i$$

$$x \ge 0$$

Now applying Lagrangian multiplier method :

$L(x, s, \lambda) = f(x) + \sum \lambda_i h_i(x)$

$= f(x) - \sum \lambda_I (g_i(x) + s_i^2 - b_i)$

Necessary condition: $\frac{\partial L}{\partial x} = 0 \Rightarrow \frac{\partial f}{\partial x} - \sum \lambda_i \frac{\partial g_i}{\partial x} = 0$ … (1.4)

$\frac{\partial L}{\partial \lambda_i} = 0 \Rightarrow g_i(x) + s_i^2 - b_i = 0 \Longrightarrow s_i^2 = b_i - g(x)$ … (1.5)

$\frac{\partial L}{\partial s_i} = 0 \Rightarrow \lambda_i s_i^2 = 0 \Longrightarrow \lambda_i (b_i - g_i(x)) = 0$ (from 1.5)

$\Longrightarrow \lambda_i = 0$ or $b_i = g_i(x)$

Given that λ_i calculates the rate at which f varies in relation to b_i.

Hence $\frac{\partial f}{\partial b_i} = \lambda_i$ and $\lambda_i s_i = 0 \Longrightarrow \lambda_i = 0$ or $s_i = 0$ or both zero.

Case 1: $s_i \neq 0 \Longrightarrow \lambda_I = 0$

Since $g_i(x) + s_i^2 = b_i \Longrightarrow g_i(x) \leq b_i$

Hence the constraint is satisfied as strict inequality. Therefore if we make the constraint larger stationary point will not be affected.

Case 2: $\lambda_i \neq 0 \Longrightarrow s_i = 0$

$\Longrightarrow$ Constraints are satisfied as equality

Let $\lambda_i < 0$, $\frac{\partial f}{\partial b_i} < 0$. Hence if b_i increases , the OF decreases . As b_i decreases, more space becomes feasible and OF cannot decrease.

$\Longrightarrow \lambda_i > 0$ or $\lambda_i = 0$

$\Longrightarrow$ *optimal solition occurs as* $\lambda_i \geq 0$.

Similarly for minimum case.

Hence the necessary condition for

i) Maximization case:

Max f (x)

St

$g_i(x) \leq b$

is:

$\frac{\partial L}{\partial x} = 0$

$\lambda_i (g-b) = 0$

$\lambda \geq 0$

$g_i \leq b_i$

$\lambda_i \geq 0$

ii) Minimization case:
Min f(x)
St
$g_i(x) \leq b_i$

$\frac{\partial L}{\partial x} = 0 \Rightarrow \frac{\partial f}{\partial x} - \sum \lambda_i \frac{\partial g_i}{\partial x} = 0$

$\lambda_i (g-b) = 0$

$g_i \leq b_i$

$\lambda_i \leq 0$

Now the sufficient conditions are:

1. Maximization case If

a) f(x) shows concavity and

b) g_i (x) be convex, then the KKT necessary conditions becomes sufficient.

2. Minimization case If

a) f(x) shows convexity and

b) g_i (x) be convex, then the KKT necessary conditions becomes sufficient.

Example: Solve the given NLPP using KKT conditions

$$\text{Max } z = 2.4 x_1 - 0.4 x_1^2 + 0.8x_2 - 0.2 x_2^2$$

Subject to

$$2x_1 + x_2 \leq 10$$
$$x_1 , x_2 \geq 0$$

Solution: $L = f(x) – \lambda g(x)$

$= (2.4x_1 – 0.4x_1^2 + 0.8x_2 – 0.2x_2^2) – \lambda(2x_1 + x_2 - 10)$

Necessary condition: $\frac{\partial L}{\partial x_1} = 0$, $\frac{\partial L}{\partial x_2} = 0$, $\lambda \geq 0$, $g \leq 0$, $X \geq 0$.

$\Longrightarrow 2.4 - 0.8x_1 - 2\lambda = 0$ ...(1)

$0.8 - 0.4x_2 - \lambda = 0$ ...(2)

$\lambda(2x_1 + x_2 – 10) = 0$ ...(3)

$\lambda \geq 0$ ...(4)

$2x_1 + x_2 \leq 10$ …(5)

$x_1, x_2 \geq 0$

Case 1: Let $\lambda = 0$

From 1 and 2 we get $x_1 = 3$, $x_2 = 4$

Altering the x_1 and x_2 values in the formula 5 we get 10 = 10. Hence x_1 and x_2 satisfies (5) also .

The stationary points are given by : $(x_1,x_2,\lambda) = (3,4,0)$

The KKT necessary condition will be met when the problem is one of maximizing, f(x) be a concave function while $g(x) \leq 0$ being a convex function.

Forming the Hessian matrix for f(x) we get $H^B = \begin{vmatrix} -0.8 & 0 \\ 0 & -0.4 \end{vmatrix}$

Minors are : $D_1 = -0.8$

$D_2 = 0.32$

Since minors D_1 and D_2 have opposite signs starting with < 0 . Hence we proved f(x) to be Concave.

Since $g(x) = 2x_1 + x_2 \leq 10$ is linear function and its all linear functions be convex. Therefore g(x) is convex and hence the KKT necessary condition is also a sufficient condition.

Therefore z = 3.6

CHAPTER 2

DUALITY IN NONLINEAR PROGRAMMING PROBLEM

Duality is a crucial idea in both linear and nonlinear problems. It is significantly simpler to solve a dual problem than a primal problem, and duality helps with convergence, finding the near-optimal solution, sensitivity analysis of the primal problem, and many other things. [4].

2.1 The canonical primal (CP) and canonical dual (CD) problems:

The CP problem of X is as follows if X be nonempty convex set and all of its functions are convex.:

$$\begin{aligned} &\text{Min } f(x) \\ \text{st} \quad & \qquad\qquad\qquad\qquad (P) \\ &g(x) \leq 0, \qquad x \in X \end{aligned}$$

where $g(x) = (g_1(x), \ldots ,g_m(x))^t$, and function f and g_i's are all real-valued defined on $X \subseteq R^n$. Next, its Dual in relation to the g-constraint is provided by:

$$\text{Maximize } [\inf f(x)+ u^t g(x) , u \geq 0 \qquad (D)$$

where u is a dual variable m-vector.

2.2 Lagrange Multiplier:

The dual is represented by transforming the provided problem into a standard Lagrange multiplier problem. We are restricting our discussion to convex case only. Let us consider real valued function f , g_i (i=1 to m) which are convex functions over $\mathbb{R}^n$.

Consider the following LPP:

$$\begin{aligned} &\text{Min } f(x) \\ \text{s.t} \quad & \qquad\qquad\qquad \ldots (2.1) \\ &g_i(x) \leq 0 \qquad i =\{1 \text{ to } m\} \end{aligned}$$

Lagrange function is given by :

$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i \mathbf{g}_i(x)$

$= f(x) + \lambda^t g(x) \qquad X \in R^n$ **,** $g(x) = (g_1(x), \ldots, g_m(x))^t$

To construct dual for LPP, we construct the following functions.

$L^*(x) = \text{Max } L(x, \lambda), \lambda \geq 0$ be the primal function.

$L_*(x) = \text{Min } L(x, \lambda)$ be the dual function.

$L^*(x)$ can be written as:

$L^*(x) = \text{Max } L(x, \lambda) \qquad \lambda \geq 0$

$= \text{Max } (f(x) + \lambda^T g(x)$

$$= f(x) = \begin{cases} f(x), if\, g_i(x) \leq 0 \\ + infinity\ , otherwise \end{cases}$$

Therefore, the problem

$\text{Min } L^*(x) = \text{Min Max } L(x, \lambda)$, is same as initial problem.

Similarly,

$\text{Max } L_*(\lambda)$, $= \text{Max Min } L(x, \lambda)$, $\lambda \geq 0$

The two problems Min Max $L(x, \lambda)$ and Max Min $L(x, \lambda)$ are related exactly the similar to how standard LPP and its dual are connected. And could be taken as standard primal-dual pair of LPP. Therefore, for the original problem, the dual is represented as:

$\text{Max } L_*(\lambda)$

$\text{Max Min } L(x, \lambda)$

$\text{Max Min } (f(x) + \lambda^T g(x))$,

$\text{Max } f(u) + \lambda^T g(u)$

st

$f(u) + \lambda^T g(u) = \text{Min}(f(x) + \lambda^T g(x)) \quad , \lambda \geq 0$

NOTE: To be more precise replace 'max' and 'min' by 'sup' and 'inf' respectively

As f and g_i are differentiable convex functions. Therefore, $L(x, \lambda) = f(x) + \sum_{i=1}^{n} \lambda_i \mathbf{g}_i(x)$ is also convex in x $\forall$ fixed , $\lambda \geq 0$. Therefore, $\nabla_x L(x, \lambda)|_{(\bar{x}, \bar{\lambda})} = 0$ iff $L(\bar{x}, \bar{\lambda})$ is the min value of $L(x, \lambda)$ i.e.

$L(\bar{x}, \bar{\lambda}) = \text{Min } [f(x) + \sum_{i=1}^{m} \lambda_i \mathbf{g}_i(x)]$

Hence the Dual is

$\text{Max } \ f(u) + \sum_{i=1}^{m} \lambda_i \mathbf{g}_i(u)]$

s.t. …(2.3)

$\nabla f(u) + \sum_{i=1}^{m} \lambda_i \mathbf{g}_i(u) = 0$

$\lambda_i \geq 0$, $i \in I$

Let eq^n (2.1) be CP and (2.2) be CD.

2.3 Theorem: Weak Duality Theorem:

Assuming x be feasible for CP and (u,λ) is feasible for CD then:

$$f(x) \geq f(u) + + \sum_{i=1}^{n} \lambda_i \mathbf{g}_i(u)$$

2.4 Wolfe's Dual

Dual problem having differentiable OF and constraints is called Wolfe duality. It was named after Philip Wolfe. We can find the lower bound for min problem because of weak duality principle using this concept. We Assumed to be convex are functions f(x) and gi(x).

$$\text{Min } f(x)$$

st

$$g_i(x) \leq 0 \qquad i = 1 \text{ to } m$$

Its Lagrangian dual (LD) problem becomes:

$$\text{Max inf } \left(f(x) + \sum_{j=1}^{m} u_j \, g_j(x) \right)$$

st

$$u_i \geq 0 \qquad i = 1 \text{ to } m$$

provided that f and g_i 's functions are convex and continuous differentiable , infimum occurs where gradient = 0

Therefore ,

$$\text{Max } f(x) + \sum_{j=1}^{m} u_j \, g_j(x)$$

st

$$\nabla f(x) + \sum_{j=1}^{m} u_j \nabla g_j(x) = 0 \qquad \ldots (2.3)$$

$$u_i \geq 0 \qquad i = 1 \text{ to } m$$

The Wolfe dual problem is the name given to the aforesaid problem. The KKT conditions are utilized as a constraint in this problem , also its inequality constraint.

$\nabla f(x) + \sum_{j=1}^{m} u_j \nabla g_j(x)$ is nonlinear in general

The Wolfe dual problem could be an optimization problem that is not convex.

Example: Write the Wolfe dual of following NLP:

Min z = $3x_1^2+6x_2$

s.t.

$x_1^2 + x_2^2 \leq 3$

$x_1 + x_2 \leq 8$

$x_1 , x_2 \geq 0$

Solution: Given $f(x) = 3x_1^2 + 6x_2$

$g_1(x) = x_1^2 + x_2^2$

$g_2(x) = x_1 + x_2$

Dual is Max $f(x) + u_1 g_1(x) + u_2 g_2(x)$
st

$$\begin{bmatrix} 6x_1 \\ 6 \end{bmatrix} + u_1 \begin{bmatrix} 2x_1 \\ 2x_2 \end{bmatrix} + u_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$u_1, u_2 \geq 0$

$x_1, x_2 \in X$

Therefore Wolfe Dual is:

Max $z = 3x_1^2 + 6x_2 + u_1(x_1^2 + x_2^2) + u_2(x_1+x_2)$

st

$6x_1 + 2x_1u_1 + u_2 = 0$

$6 + 2x_2u_1 + u_2 = 0$

2.5 Lagrange Dual (LD):

We examine optimization problems constrained by inequality as well as equality.

Minimize $f(x)$

st

$h_i(x) = 0, \quad i = 1, \dots, m,$

$g_j(x) \leq 0, \quad j = 1, \dots, r$

Let f* be the function f 's optimal value under the constraints; that is, $f^* = f(x^*)$ if the minimum is reached at x*, the global minimum of the function.

2.5.1 Lagrange dual function:

The Lagrangian of aforementioned problem is the function L: $R^n \times R^m \times R^r \to R$ defined by: $L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{j=1}^{m} \lambda_j g_j(x)$

Now the LD function q: $R^m \times R^r \to R$ is defined by:

$q(\lambda, \mu) = \inf L(x, \lambda, \mu)$, $x \in R^n$

$= \inf (f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{j=1}^{m} \lambda_j g_j(x))$

Therefore, regarding the specified primal problem:

Minimize $f(x)$

st

$h(x) = 0$

$g(x) \leq 0$

Its LD problem is:

Maximize $q(\lambda, \mu)$

St

$\mu \geq 0$

q being a concave LD function in this case and its Lagrange multipliers are μ associated with the constraints $g(x) \leq 0$ and λ associated with $h(x) = 0$.

2.6 Geometric Interpretation

We examine the primal problem:

Minimize f(x)

st

$g(x) \leq 0,$

where functions f , g : $R^n \rightarrow R$. We shall explain its weak and strong duality geometrically.

a) Optimal value f*

Examine the R^2 subset that is specified by $S = \{ (g (x), f (x) \mid x \in \mathbb{R}n) \}$.The formula yields the optimal value f*.:

$$f^* = \inf \{ t \mid (u, t) \in S, u \leq 0 \}$$

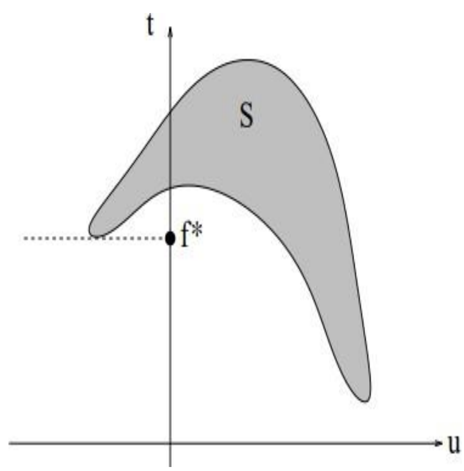

Figure 2.1: Optimal value f*

b) Dual function q(μ)

For $\mu \geq 0$, the dual function is provided by:

$$q(\mu) = \inf \{ f(x) + \mu\, g(x) \} , x \in \mathbb{R}^n$$

$$= \inf \{\mu u + t\} \qquad , (u,t) \in S$$

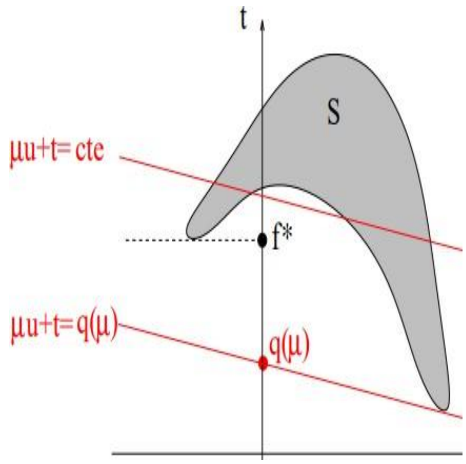

Figure 2.2: Dual function

c) Dual optimal d*

The dual optimum value is provided by:

$$d^* = \sup q(\mu) \;,\; \mu \geq 0$$
$$= \sup \inf\{\mu u + t\}, (u,t) \in S$$

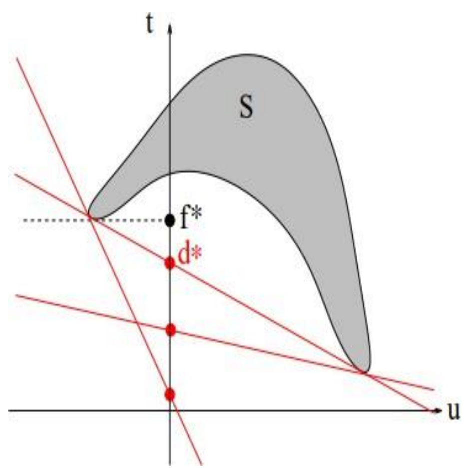

Figure 2.3: Optimal of dual

d) Weak duality

$d^* \leq f^*$

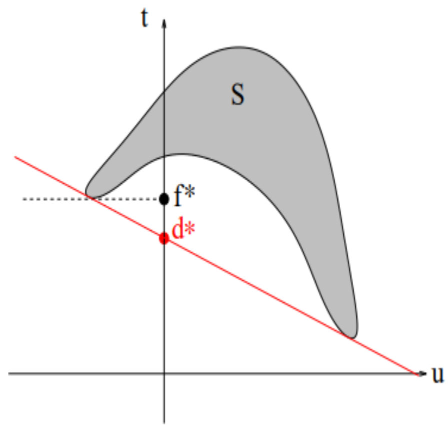

Figure 2.4: Weak duality

e) Strong duality:

For convex problems when the points are strictly feasible: d* = f*

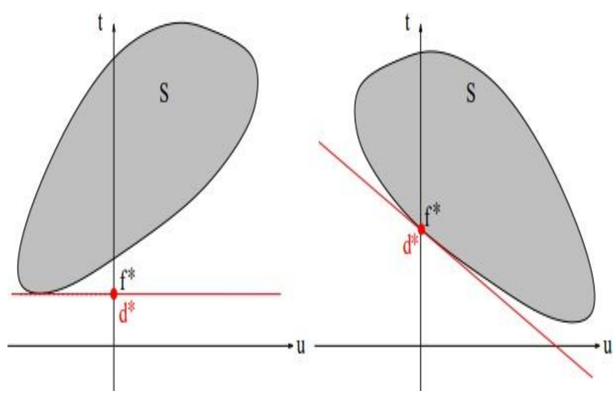

Figure 2.5: Strong Duality

CHAPTER 3

ECONOMIC INTERPRETATION OF DUALITY

In economics, nonlinear problems often involve non-proportional relationships between variables. The economic interpretation depends on the specific context, but nonlinearities can capture phenomena such as diminishing returns, increasing marginal costs, or complex market dynamics. Analyzing these nonlinearities allows for a more realistic representation of economic systems, enabling better policy insights and decision-making.

3.1 Investment Analysis

An important function of financial management is efficient allocation of funds. Investment analysis consist of evaluation of cash flows, consideration of proposals for investment.

3.2 Portfolio Model

Now we will discuss economic interpretation of duality using the portfolio model which aims to maximize the return and minimize the risk. A portfolio model is a framework that is used in finance and investment management to build and oversee a group of assets (bonds, stocks, and other securities) in order to accomplish particular financial goals.

3.2.1 The Primal-Dual Interior Point Method for Solving Nonlinear Portfolio Optimization Problem [5]

Stochastic programming is recognized as a powerful tool in financial planning for facilitating decision-making amidst uncertainty. Large dimensions characterize the deterministic equivalent formulations of stochastic systems, even when considering modest numbers of assets, time phases, and outcomes per time. Until now, approaches to mathematical programming have only been able to handle basic linear or quadratic models because the solvers that are currently on the market cannot handle NLP issues with typical sizes. Stochastic programming issues, however, have a lot of structure. Therefore, being able to take advantage of their structure is essential to solving such challenges effectively. Large-scale nonlinear optimization problems are a good fit for interior point approaches. In this study, we leverage this capability to illustrate how the most recent solver can be employed to address POP involving millions of decision variables and constraints along with constrains on the objective's skewness, semi-variance, or nonlinear utility functions. We show that the mathematical programming approach can deal with the huge problems arising from portfolio optimization, even when extensive nonlinear constraints or objective terms are present. Markowitz [6,7] mean variance model may lead to inferior conclusions, i.e some optimal solutions may be stochastically dominated by other feasible solutions.[8,9]. Therefore Ogryczak and Rusczynski [8] replaced the usual objective in the Markowitz model with a new one composed of mean and semi deviation. Additionally, they contend that in a case of an asymmetric distribution of asset returns, investors' preference for a positive deviation from the mean should be expressed by adding skewness to the OF. Changes to the Markowitz model [6,7] need adding nonlinearities to the constraints or the objective function, It yields formulations that are deterministic counterparts of the class NLP (nonlinear programming).

The paper's goal is to show that solving very large, complex, nonlinear stochastic algorithms with a modern general structure that uses an interior point approach—like our Object-Oriented Parallel Solver (OOPS)—is feasible. [10,11]. Moreover, this approach can be used with a variety of objective functions and constraints in stochastic programming and is not restricted to any particular environment.

3.2.1(a) Mean-Variance problem with Asset Liability Management (ALM)

We will offer a multi-step procedure for managing assets and liabilities that uses variance to evaluate risk and mean to determine return. Our problem description is in accordance with those found in [12,13]. The best strategy to invest in assets with j = 1,...,J is what we are trying to figure out. Asset returns are unpredictable. The portfolio may need to be rebalanced periodically. t = 1,...,T after an initial cash investment of b at t = 0. Maximizing the expected ultimate portfolio's value at time T + 1 and reducing related risk as shown by the final wealth variance are the goals. By x_t , we indicate the decision variables at time t.

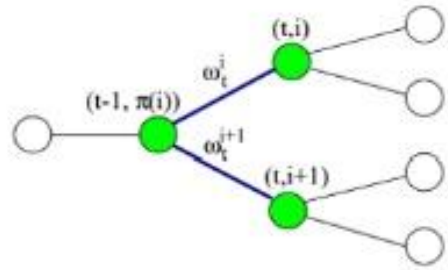

Figure 3.1: Event tree of portfolio model

An event tree can be used to illustrate the process's unpredictable nature. Discrete random occurrences ω_t with, at moments t = 0, There are only a limited amount of outcomes that can occur t = 0 . . . , T. For any sequence of observed events (ω_0, ..., ω_t), we anticipate that the next observation, ω_t + 1, will provide one of a finite number of alternative outcomes. A tree with its roots at the original event ω_0 is produced by this branching process. At time t, Lt stands for the collection of nodes of past observations L_T represents the final nodes set and indicate the set of all nodes as $L = \cup_t L_t$, the collection of end nodes (leaves)

From here on, a variable $i \in L$ will represent nodes; as seen in Fig.3.1, i = 0 will represent the root node, and π(i) will represent the node i's antecedent (parent). Moreover, node i+ 1 before π(i).

Pi represents the probability of reaching node I, which is calculated as the sum of the probabilities for each level set that adds upto give one. Given an asset j and a transaction cost c_t , let vj be its value.One can always purchase an asset j for $(1 + c_t)v_j$ or sell it for $(1 – c_t)v_j$, as it is expected that the asset's value would remain constant over time.

An additional return $r_{i,j}$ will be generated by an asset j unit stored in node i in its place. (originating from node π(i)). $x^h_{i,j}$ denotes the quantity of assets j held at node i in units and the number of transactions (buying, selling) of this asset at this node (i) is denoted by $x^b_{i,j}$, $x^s_{i,j}$, respectively. In a similar vein, the random variables $x^h_{t,j}$, $x^b_{t,j}$, and $x^s_{t,j}$ describe the holding, purchasing, and selling of asset j at time stage t. We make the assumption that we have b dollars at the start to invest but zero holdings in all assets. Additionally, cash is assumed to be one of the assets, and available funds are assumed to be completely invested at all times . In actuality, the value of asset v_j fluctuates in response to market swings, whereas the quantity of asset $x^h_{t,j}$ stays constant (until buying or selling occurs). Set a modelling standard that treats each asset's wealth accumulation as a product $v_j \cdot x^h_{t,j}$, where $x^h_{t,j}$ changes and v_j remains constant.

The following is an expression of the usual investment policy constraints: Cash balance limitations, which take transaction costs into consideration, outline potential purchasing and selling actions inside a scenario:

$$\sum_j (1 + c_t) v_j \, x^b_{i,j} = \sum_j (1 - c_t) v_j \, x^s_{i,j} \forall \; i \neq 0 \qquad \ldots (3.1)$$

$$\sum_j (1 + c_t) v_j \, x^b_{0,j} = b$$

Inventory constraints, which are balancing restrictions on asset holdings that take into consideration the erratic return on asset, bind each scenario to its parent:

$$(1 + r_{i,j}) \, x^h_{\pi(i),j} = x^h_{i,j} - x^b_{i,j} + x^s_{i,j} \;\; \forall \; 0, j \qquad \ldots (3.2)$$

Non negative variables are:

$$x^h_{i,j} \geq 0, \; x^b_{i,j} \geq 0 \;, \; x^s_{i,j} \geq 0, \; \forall \; i,j \qquad \ldots (3.3)$$

The ultimate wealth maximization and the reduction of related risk are taken into consideration. The final portfolio's anticipated value transformed into cash is how the final wealth, y, is simply represented [14]

$$y = E \left((1 - c_t) \sum_{\mathbf{j=1}}^{\mathbf{J}} \mathbf{v}_j \, x^h_{T,j} \right)$$

$$= (1 - c_t) \, \Sigma_{i \in LT} \; p_i \sum_{\mathbf{j=1}}^{\mathbf{J}} \mathbf{v}_j \, x^h_{i,,j} \qquad \ldots (3.4)$$

Traditionally, risk is represented as the return's variance:

$$\mathrm{Var} \left((1 - c_t) \sum_{\mathbf{j=1}}^{\mathbf{J}} \mathbf{v}_j \, x^h_{T,,j} \right) = \Sigma_{i \in LT} \, pi \, [(1 - c_t)] \sum_j v_j \, x^h_{i,j} - y] \quad \ldots (3.5)$$

The traditional Markowitz POP combines these two goals into a single objective that takes the form.

$$f(x) = E(X) - \rho \, \mathrm{Var}(X) \qquad \ldots (3.6)$$

Total worth of the portfolio when converted to cash (3.4) is denoted by X, and the scalar ρ represents the investor's risk tolerance. Accordingly, we would maximize (3.6) in the classical model while taking into account the constraints (3.1), (3.2), and (3.4). Convex quadratic programs are problems like this. Concave quadratic function of x is its objective, and all of its restrictions are linear. When variance (3.5) accumulates all $x^h_{T,j}$

(assets held in the previous time period), a quadratic form x^TQx is generated, where matrix Q contains large number of nonzero entries.Rephrasing the problem to take advantage of the variance's partial separability was suggested by Gondzio and Grothey [10]. This would yield a non-convex formulation of the problem, albeit with a significantly increased sparsity in the quadratic term.
The reformulated expression utilizes an alternative variance representation ($Var(X) = E(X^2) – E(X)^2$) :

$$Var((1 - c_t) \sum_{j=1}^{J} \mathbf{v}_j x^h_{T,j}) = E((1 - c_t)^2 [\sum_j v_j x^h_{T,j}]^2) - E((1 - c_t) \sum_j v_j x^h_{T,j})^2$$

$$= \Sigma_{i \in LT} \; p_i (1 - c_t)^2 [\sum_j v_j \; x^h_{T,j}]^{2} - y^2$$

Remarkably, We find that the space of constraints is convex in our non-convex formulation. While one extra choice variable, y, is used in the goal in the non-convex formulation, the resulting Hessian matrix has improved block-sparsity qualities.

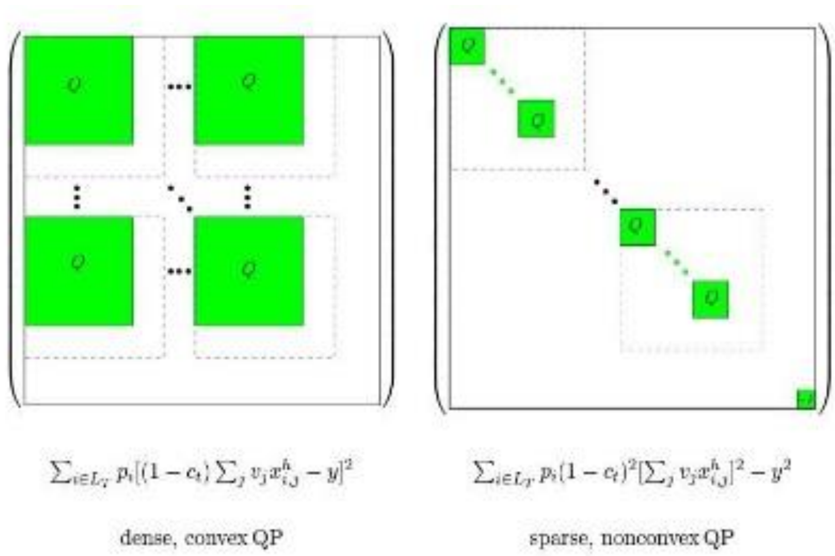

Figure 3.2 Hessian Matrix in two compositions of the variance

3.2.1(b) Extensions and solution of ALM problems

The serious drawback of classical Markowitz model was that the the mean-variance model fails to meet the requirements for second-order stochastic dominance.
We can easily extend the previous section to consider only a semi-variance (downside risk).
To improve the flexibility of modelling , we include a constraint and two additional

variables, s_i^+ and s_i^- per node $i \in L_T$,illustrating the deviations from the mean, both positive and negative.

$$s_i^+ \geq 0 \,,\, s_i^- \geq 0, \forall\, i \qquad \ldots (3.7)$$

$$(1 - c_t) \sum_{j=1}^{J} \mathbf{v}_j\, x^h_{i,j} + s_i^+ - s_i^- = y,\ \ i \in L_T \qquad \ldots (3.8)$$

It's possible that not every situation mentioned calls for these variables. Their main goal is to demonstrate how simple it is to incorporate additions to the mean-variance model, which results in structured sparse problems.

$$\mathrm{Var}(X) = \Sigma_{i \in LT}\ p_i(s^+_i - s^-_i)^2 = \Sigma_{i \in LT}\ p_i((s^+_i)^2 + (s^-_i)^2) \,,\ \ldots (3.9)$$

The Hessian matrix's sparsity would be further enhanced by the formulation that includes these slack variables. The Hessian Matrix formed would be diagonal and hence the QP will be separable . This new formulation makes it simple to extend the method of measuring downside risk by using the semi variance $E[(X - X)^2]$.

$$\mathrm{sVar}(X) = \Sigma_{i \in LT}\ p_i(s_i^+)^2 \qquad \ldots (3.10)$$

$$\begin{aligned} &\mathrm{Max}_{x,y,s>0}\ \ y - \rho\, [\, \Sigma_{i \in LT}\ p_i\, ((s_i^+)^2 + (s_i^-)^2)] \\ \text{s.t} \qquad & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \ldots (3.11) \\ &(3.1), (3.2), (3.3), (3.4), (3.7), (3.8) \end{aligned}$$

which is a convex QPP.

The following modifications, which result in formulations of nonlinear problems, will be discussed:

1. A risk constraint (assessed by variance): We directly represent the risk-adversity of investors using nonlinear constraints. , i.e.

$$\begin{aligned} &\mathrm{Max}_{x,y,s\geq 0}\ \ [\, \Sigma_{i \in LT}\ p_i\, ((s_i^+)^2 + (s_i^-)^2) \leq \rho] \\ \text{s.t} \qquad & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \ldots (3.12) \\ &(3.1), (3.2), (3.3), (3.4), (3.7), (3.8) \end{aligned}$$

2. A constraint on downside risk (assessed by the semi-variance):

$$\begin{aligned} &\mathrm{Max}_{x,y,s\geq 0}\ \ y \\ \text{s.t} \qquad & [\, \Sigma_{i \in LT}\ p_i(s_i^+)^2 \leq \rho] \qquad \ldots (3.13) \end{aligned}$$

s.t (3.1) ,(3.2) ,(3.3) ,(3.4) ,(3.7) ,(3.8)

3. The objective includes a logarithmic utility function:

$$\text{Max}_{x,y,s\geq 0} \quad (1 - c_t) \Sigma_{i\in LT} \; p_i \log(\textstyle\sum_{j=1}^{J} \mathbf{v}_j \, x^h_{i,j})$$

s.t $\Sigma_{i\in LT} \, p_i \, ((s_i^+)^2 \leq \rho$ … (3.14)

(3.1) ,(3.2) ,(3.3) ,(3.4) ,(3.7) ,(3.8)

4. An objective having skewness:

$$\text{Max}_{x,y,s\geq 0} \quad y + \Sigma_{i\in LT} \, p_i \, (s_i^+ - s_i^-)$$

s.t $\Sigma_{i\in LT} \, p_i \, ((s_i^+)^2 + (s_i^-)^2 \leq \rho$ … (3.15)

(3.1) ,(3.2) ,(3.3) ,(3.4) ,(3.7) ,(3.8)

All these formulations are NLPP.

We apply a SQP approach to assess the nonlinear ALM variations (3.12) to (3.15). In this method, our OOPS serves as the primary QP solver. An SQP algorithm to solve

$$\text{Min} \quad f(x)$$

s.t

$$g(x) \leq 0,$$

generates primal iterates ($x^{(k)}$) and dual iterates ($\lambda^{(k)}$) and solves the QPP at each step

$$\text{Min}_{\Delta x} \; \nabla f(x^{(k)})^T \Delta x + 1/2 \; \Delta x^T \, Q \Delta x$$

s.t

$$A\Delta x \leq -g(x^{(k)}) \qquad \text{…(3.16)}$$

where $A = \nabla g(x^{(k)})$ is the Jacobian of the constraints and $Q = \nabla^2 f(x^{(k)}) + \sum_i \lambda^{(k)} \nabla^2 g_i (x^{(k)})$ being the Lagrangian's Hessian**.** OOPS, our interior point based QP solver, is capable of effectively utilising almost any nested block structure that exists between the system matrices A and Q.

We have demonstrated how to use general purpose structures to take use of interior point solvers to solve nonlinear portfolio optimization issues. Three modifications to the conventional mean-variance approach to the Asset and Liability Management problem have been the focus of our attention; these modifications generate distinct nonlinear programming problems.

Although the theoretical significance of these variations has been acknowledged for some time, the conventional thinking holds that mathematical programming techniques are not appropriate for these models. We have demonstrated that this is no longer the case in light of recent developments in structure that take advantage of interior point solvers. Currently, nonlinear ALM problems with millions of variables can be handled by a general solver in a reasonable amount of time. These problems can be solved by standard optimization software by employing a structure exploiting interior point method

CHAPTER 4

CONCLUSION

We have presented an example of economic interpretation of duality of nonlinear programming by presenting a portfolio model. We have discussed 2 methods of solving NLPP, the first one being Lagrange Multiplier Method and the second one KKT conditions. We have also discussed The duality of NLPP and subsequently we have discussed Wolfe dual and Lagrange dual briefly. An instance of utilizing an interior point solver with a general purpose structure to tackle nonlinear POP was also discussed.

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