NONLINEAR RANDOM VIBRATION ANALYSIS USING TAIL EQUIVALENT LINEARIZATION METHOD

Thesis submitted in partial fulfilment of the requirements for award of the degree of

MASTER OF TECHNOLOGY

In

STRUCTURAL ENGINEERING

Submitted by

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UNDER THE SUPERVISION OF

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CANDIDATE'S DECLARATION

I hereby declare that the project work entitled **"**NONLINEAR RANDOM VIBRATION ANALYSIS USING TAIL EQUIVALENT LINEARIZATION METHOD**"** submitted to Department of Civil Engineering, DTU is a record of an original work done by **KUMAR VISHAM** under the guidance of **Mr. G.P.Awadhiya,** Associate Professor, Department of Civil Engineering, DTU, and this project work has not performed the basis for the award of any Degree or diploma/fellowship and similar project, if any.

KUMAR VISHAM
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CERTIFICATE

This is to certify that the project entitled **“**NONLINEAR RANDOM VIBRATION ANALYSIS USING TAIL EQUIVALENT LINEARIZATION METHOD**”** submitted by **KUMAR VISHAM,** in partial fulfillment of the requirements for award of the degree of **MASTERS OF TECHNOLOGY (STRUCTURAL ENGINEERING)** to Delhi Technical University is the record of student’s own work and was carried out under my supervision.

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Last but not the least I thank my parents, for everything I am and will be in future.

ABSTRACT

A new non-parametric linearization method for nonlinear random vibration analysis is created. This method works on a discrete representation of the stochastic inputs and the ideas from the first order reliability method (FORM). For a specified response threshold of the nonlinear system, the equivalent linear system is characterized by matching the "design points" of the linear and nonlinear responses in the space of the standard normal variables acquired from the discretization of the excitation. Because of this definition, the tail probability of the linear system is equal to the first order approximation of the tail probability of the nonlinear system, this property motivating the name Tail-Equivalent Linearization Method (TELM).This leads to the identification of the TELS in terms of a unit-impulse response function for each component of the input excitation,tail equivalent linearization method is a new,non-parametric linearization method for nonlinear random vibration analysis.This method is to overcome the inadequacy of conventional equivalent linearization method.Our objectives are investigation and thorough understanding of analysis of stochastic non-linear system by tail equivalent linearization method as well as computation of certain nonlinear response characteristics. Further more study is presented on method of random vibrational analysis especially on equivalent linearization method and also gives brief review on reliability analysis of structure, first order reliability analysis (FORM).It is demonstrated that the equivalent linear system is determined in terms of its impulse response function in the non-parametric form from the knowledge of design point. This examination looks at the impacts of different parameters on the tail-equivalent linear system, presents an algorithm for finding the design points. Design point in FORM is the point on a limit-state surface that is nearest to the origin when the random variables are transformed to the standard normal space.Linearization of the limit-state surface at this point uniquely defines a linear system, denoted as Tail-Equivalent Linear System, TELS.Previous study shows that design point shows that design point on limit state surface of linear system and nonlinear system is same. Once the TELS is defined for a specific response threshold, methods of linear random vibrational analysis are used to compute various response statistics, such as the mean crossing rate and tail probabilities of local and extreme peaks. The method has been developed for application in both time, and frequency domain and it has been applied to inelastic structures as well as structures experiencing geometric nonlinearities.

LIST OF FIGURES

CHAPTER 1
INTRODUCTION

1.1 General

Stochasticity and non-linearity are characteristics of many structural and mechanical engineering problems. In some sense, we can say that they are mother of all problems. They are essential considerations in assessing the reliability of structural and mechanical systems under extreme loads, e.g.

- Inelastic structural response to strong earthquake ground motion.
- Response of offshore structures to wave loading under material and geometrical non-linearity's.
- Response to turbulent winds.

Existing methods of nonlinear stochastic dynamic analysis are restricted to special cases or are not suitable for reliability analysis-hence the need of the new method.

In evaluating the safety of a structure, it is imperative to incorporate the nonlinearity, since failure generally happens in the nonlinear range of structural behaviour.

The topic of nonlinear random vibration has been the focus of much research and development in the previous several decades Methods developed include the Fokker–Planck equation, stochastic averaging, moment closure , perturbation, and equivalent linearization .Among these, the equivalent linearization method has gained wide popularity because of its versatility in application to general, multi-degree-of-freedom nonlinear systems.The other methods, possibly more accurate , are largely restricted to specìalized systems or forms of the excitation, and are difficult to apply in practice. The Monte Carlo simulation method is without restriction, but is computationally demanding.

In the equivalent linearization method (ELM), the nonlinear system of interest is replaced by an equìvalent linear system, the parameters of which are determined by minimizing a measure of the discrepancy between the responses of the nonlinear and linear systems . The measure of discrepancy most often used is the mean-square error between the two responses ,although an energy-based measure has also been considered. The solution requires an iterative scheme, since the parameters of the linear system are functions of the second-moments of its response. Furthermore, the method requires an assumption regarding the probability distribution of the nonlinear response and most often the Gaussian distribution is selected. As a result, while the method can be quite accurate in estimating the mean-square response, the probability distribution can be far from correct, particularly in the tail region. It follows that estimates of such response statistics as crossing rates and first-passage probability, which are of particular interest in reliability analysis, can be grossly inaccurate at high thresholds. To address this problem, an alternative linearization method was proposed by Casciati by equating the mean level crossing rates of the nonlinear and equivalent linear systems. However, this approach requires knowledge of the joint probability distribution of the response and its derivative, which can be extremely difficult to obtain for general nonlinear systems, particularly those having multiple degrees of freedom.

The method proposed in this study is also an equivalent linearization method. However, instead of defining the linear system by minimizing the mean-square error in the response, it is defined by matching the tail probability of the linear response to a first-order approximation of the tail probability of the nonlinear response .For this reason, thezname Tail-Equivalent Linearization Method (TELM) is used. The genesis of the method lies in the first-order reliability method (FORM) and the earlier works of Li and Der Kiureghian, Der Kiureghian and Koo. This study formalizes the method and investigates the various characteristics of the tail-equivalent linear system (TELS).

After describing a method for discrete representation of the stochastic excitation ,geometric characteristics of a linear system in the space of standard normal random variables are examined. It is shown that a reversible relationship exists between the impulse response function of the system and the gradient vector of a hyperplane defining a threshold of interest. This then leads to a formal definition of the TELS for a general nonlinear system. Issues related to the existence and uniqueness of the TELS and the influences of various key parameters on the TELS are examined. An algorithm for finding the sequence of linearization points necessary for determining the full probability distribution of the response is next described, followed by a discussion of methods for determining various response statistics. Throughout the study results are presented for a hysteretic oscillator and, where appropriate, comparisons are made between results obtained by the TELM and the conventional ELM.

1.2 Objective and scope of the study

The objectives are investigation and thorough understanding of analysis of non-linear system by Tail Equivalent method as well as computation of certain non-linear response characteristics. Proper algorithm for finding the design point has been presented.

This study to present thorough investigation of nonlinear stochastic dynamic analysis using TELM (Tail Equivalent Linearization Method), and influence of various parameters on the tail equivalent linear system, such as discrete representation of stochastic excitation, characterization of linear system etc. Apart from TELM for the use of white noise Gaussian process. For studying of TELM we want basic idea about random vibration analysis and methods of structural reliability analysis.

TELM is based on first order reliability method and equivalent linearization method of random vibration.

TELM is combination of FORM and ELM means reliability analysis and random vibration analysis. In this study we give brief review of both the methods.

The method was initially developed in the field of earthquake engineering, where a discretization in time domain is convenient. A corresponding definition of the tail equivalent linearization system was then obtained in terms of its unit impulse-response function.

A number of applications of this method in civil engineering filed has been investigated for both stationary and non-stationary problems,single and multi degree of freedom systems,and a variety of non-degrading,hysteretic material models,demonstrating its validity and accuracy.

1.3 Organization of Report

This report is organized into twelve chapters.

In the first chapter a short review of TELM and importance of this method are given..For understanding TELM, we require a good knowledge of random vibration analysis and reliability analysis of structure so that we also require a review of both method of analysis.

The second chapter is literature review in which the works of previous scientists on Tail equivalent linearization method are explained.

In chapter three, the different methods of nonlinear stochastic analysis are overviewed. This includes classical methods, simulation methods and linearization methods.

In chapter four, the characteristics of linear system are explained.

In chapter five, we study about reliability. Various terminologies used in reliability are defined. We study about normal distribution function. And finally first order reliability method is studied. Non-linear system is transformed into equivalent linear system and we calculate the design point and further reliability index is calculated.

In chapter six, the various steps to discretize nonlinear stochastic process is explained. This includes time-domain discretization and frequency-domain discretization.

In chapter seven, we see how to use FORM to solve stochastic dynamic problems.

In chapter eight, we study how to identify linear system in time-domain and in frequency domain.

In chapter nine, we study about the Tail Equivalent Linearization Method. A brief introduction of TELM is given. Then the various steps in TELM are explained. Then the iterative algorithms to find the design point is shown.

In chapter ten, we study about the various characteristics of the tail equivalent linearization method. A numerical problem to show the various characteristics of TELM is used by using a SDOF inelastic hysteretic oscillator based on Buoc Wen Model. Before we solve the above problem, we should know the different methods which are used to evaluate dynamic response. We solve a numerical example given in A.K.Chopra book by linear interpolation and Newmark’s method.

In chapter eleven, we study about the shortcomings and limitations of TELM and finally

In chapter twelve, we obtain the conclusions from the whole project.

CHAPTER 2

LITERATURE REVIEW

Kazuya Fujimara,Armen Der Kiureghian , presented tail equivalent linearization method which uses the advantages of first order reliability method(FORM).In this method,stochastic excitation is discreticized and represented in terms of finite set of standard normal variables.TELM is new,non-parametric linearization method for nonlinear random vibration analysis.For a specified response of threshold of the nonlinear system.The equivalent linear system is defined by matching the "design points" of the linear and nonlinear responses in the space of the standard normal random variables obtained from the discretization of the excitation.Due to this definition,the tail probability of the linear system is equal to the first-order approximation of the tail probability of the nonlinear system,for this property motivating the name Tail-Equivalent Linearization Method(TELM).He has shown that the equivalent linear system is uniquely determined in terms of its impulse response function in a non-parametric form from the knowledge of design point.He examined the influences of various parameters on the tail-equivalent linear system,presents an algorithm for finding the needed sequence of design points,and describes methods for determining various statistics of the nonlinear response,such as the probability distribution,mean level-crossing rate and the first-passage probability. Applications to single and multi degree of freedom,non-degrading hysteretic systems illustrate various features of the method,and comparisons with the results obtained by Monte Carlo simulations and by the conventional equivalent linearization method(ELM) demonstrate the superior accuracy of TELM over ELM, particularly for high response thresholds.

Luca Garre,Armen Der Kiureghian , extended the previous work on the Tail-Equivalent Linearization Method (TELM) to the frequency domain. The extension defines the Tail-Equivalent Linear System in terms of its frequency-response function.This function is obtained by matching the design point of the nonlinear response with that of the linearized response.The proposed approach is particularly suitable when the input and response processes are stationary,as is usually the case in the analysis of marine structures. When linear waves are considered, the Tail-Equivalent Linear System possesses a number of important properties,such as the capability to account for multi-support excitations and invariance with respect to scaling of the excitation. The latter property significantly enhances the computational efficiency of TELM for analysis with variable sea states .Additionally,the frequency-response function of the Tail-Equivalent Linear System offers insights into the geometry of random vibrations discretized in the frequency domain and into the physical nature of the response process.The proposed approach is applied to the analysis of point-in-time and first-passage statistics of the random sway displacement of a simplified jack-up rig model.A basic requirement of TELM is the discretization of the input excitation in terms of a finite set of standard normal variables. In fact,the equivalence in TELM is established in the space of these random variables by matching the design points of the linear and nonlinear responses,which are points on their respective limit state surfaces with minimal distances from the origin in the standard normal space.The method was initiallyzdeveloped in the field of earthquake engineering,where a discretization in time domain is convenient.A corresponding definition of the tail-equivalent linear system was then obtained in terms of its unit impulse-response function.A number of applications of this method in the field of civil engineering have been investigated for both stationary and non-stationary problems,single and multi DOF systems,and a variety of non-degrading,hysteretic material models demonstrating its validity and accuracy.

Armen Der Kiureghian and Kazuya Fujimura, A new alternative approach for computing seismic fragility curves for nonlinear structures for use in PBEE analysis is proposed.This approach is proposed.The approach makes use of a recently developed method for nonlinear stochastic dynamic analysis by tail-equivalent linearization.The approach avoids repeated time-history analysis with a suite of scaled,recorded ground motions.Instead,the ground motion is modelled as stochastic process and after determining TELS for each response threshold,simple linear random vibration analysis are performed to compute the fragility curve.In the present application,the same stochastic model was model was used for all intensity level to more realistically characterize high-intensity motions.In doing this,this since the TELS remains invariant of the scaling and frequency content of the excitation,one will only need to change the excitation model in the linear random vibration analysis of the TELS for different intensity levels.

While offering a viable alternative for fragility analysis,the proposed method has its limitations.For example,at the present time it is only applicable to non-degrading systems,and only one component of ground motion was considered in the present application. Furthermore,response gradient computations are required and therefore, a dynamic analysis code with this capability must be used.Nevertheless, the proposed method offers an alternative to a type of analysis for which few other viable alternatives are presently available.

Sanaz Rezaeian and Armen Der Kiureghian, described in her report stochastic modelling and simulation of ground motion time histories for use in response-history or stochastic dynamic analysis.Ultimately,this research benefits the emerging field of performance based earthquake engineering(PBEE) by providing a convenient method of generating synthetic ground motions for specified design scenarios that have characteristics similar to those of real earthquake ground motions. A new site-based,fully non stationary stochastic model to describe earthquake ground motions is developed.The model is based on time modulation of the response of a linear filter with time-varying characteristics to a discretized white-noise excitation.It is concluded that for typical strong ground motion the filter frequency can be generated by a linear function,whereas the filter damping ratio can be represented by a constant or a piece-wise constant function.

Caughey TK proposed generalized to the case of nonlinear dynamic systems with random excitation.The method is applied to a variety of problems and results are compared with exact solutions of the Fokker-Planck equation for those cases where the Fokker-Planck technique might be applied.Alternate approaches to the problem are discussed including the characteristic function.

Armen Der Kiureghian, The geometry of random vibration problems in the space of standard random variables obtained from discretization of the input processes is described.For linear systems subjected to Gaussian excitation,simple geometric forms,such as vectors,planes and ellipsoids ,characterize the problem of interest.For non-Gaussian responses,non-linear geometric forms characterize the problems.Approximate solutions for such problems are obtained by use of FORM and SORM.Examples involving response to non-Gaussian excitation and out-crossing of a vector process from a non-linear domain are used to determine the approach. Given a discrete representation of the input process in terms of standard normal variables,It is shown that many statistical quantities of interest in random vibrations can be represented in geometric form in the standard normal space.These interpretions offer a new outlook to random vibration problems and potentially provide new tools for the approximate solution of non-Gaussian or non-linear problems.In this article,solution methods by FORM and SORM were explored.Possibilities for developing efficient simulation methods that exploit the geometric forms also exist.The numerical examples presented in this indicate that FORM and SORM can be effective methods of solution,but they should be used with caution.

Heonsang Koo,Armen Der Kiureghian,Kazuya Fujimara , A key step in finding the design-point excitation,which realization of the input process that is most likely to give rise to the event of interest.It is shown that for a non-linear elastic SDOF oscillator subjected to a Gaussian white-noise input,the design-point excitation is identical to the mirror image of the free-vibration response of the oscillator when it is released from the target threshold.With a slight modification,this result is extended to problems with non-white and non-stationary excitations, as well as to hysteretic oscillators.For these cases only an approximation to the design point is obtained.If necessary the approximation can be used as a ‘warm’ starting point in an iterative algorithm to obtain the exact design point.

M.Ababneh,M.Salah,K.Alwidyan, ,in his paper,a comparison between the optimal linear model and Jacobian linearization technique is conducted.The performance of these two linearization methods are illustrated using two benchmark nonlinear systems,these are inverted pendulum system;and Duffing chaos system.Linearization of nonlinear dynamical systems.Optimal linear model is a online linearization technique for finding a local model that is linear in both the state and control terms.

Faycal Ikhouane,Victor Manosa,Jose Rodellar, The Bouc-Wen model, widely used in structural and mechanical engineering, gives an analytical description of a smooth hysteretic behaviour. It may happen that a Bouc-Wen model presents a good matching with the experimental real data for a specific input, but does not necessarily keep significant physical properties that are inherent to the real data ,independently of the exciting input.This literature presents a characterization of the different classes of Bouc-Wen models in terms of their bounded input-bounded output stability property,and their capability for reproducing physical properties inherent to the true system they are to model.

CHAPTER 3

METHODS OF NONLINEAR STOCHASTIC ANALYSIS

3.1 Introduction

Classical methods: Perturbation methods, Fokker-Plank equation, stochastic averaging, moment closure, etc.

Simulation methods: Monte Carlo Simulation (MCS), Importance Sampling (IS), Markov Chain Monte Carlo (MCMC), Latin Hypercube Sampling (LHS), Orthogonal plane sampling, etc.

Linearization methods: Classical Equivalent Linearization Method (ELM), Tail-Equivalent Linearization Method (TELM).

The classical methods are important and elegant approaches, but are limited to specialized systems or excitations. The broad family of simulation methods has no theoretical limits however, some of these methods are computationally inefficient for high reliability problems (such as most civil structures). The final class of methods offers an efficient and fairly accurate estimation of the response distribution for many structural problems. However, the standard ELM, which is a parametric method, is designed to accurately estimate the first and the second-moments of the response distribution. Since the method is not meant for estimating the tail of the distribution, it is not accurate for computing the probability of failure for highly reliable systems. The TELM is a recent linearization method based on the first-order reliability method (FORM) developed by Fujimura and Der Kiureghian. It aims at providing a good estimation of the tail probability of the nonlinear response for this class of problems.

3.2 Classical methods

3.2.1 PERTURBATION - Among the classical methods, perturbation methods are probably the first ones to be used in nonlinear random vibration. First introduced in this field by

Crandall, these are fairly general methods to solve deterministic and/or stochastic nonlinear mechanics problems.

Perturbation methods are based on power series expansion of the solution, where only "significant" terms are retained. The differential equations are formulated for each term of the expansion. The procedure is rather straightforward. However, due to the nature of the formulation, the expansion terms rapidly increase in complexity when high-order terms are considered. In addition, these methods are usually limited to lightly nonlinear systems.

3.2.2 FOKKER-PLANCK EQUATION - The Fokker-Planck equation was derived in the context of statistical mechanics, it is a partial differential equation that describes the evolution in time of the probability density function of a non-stationary process. The solution of this equation provides the exact probabilistic structure of the response at all times. However, solutions for nonlinear problems are scarce and typically are limited to situations where the response process is Markovian. Moreover, the required computational effort rapidly increases with the number of degrees of freedom of the structure.

3.2.3 MOMENT OF CLOSURE- Moment of closure is an approximate method for estimating the statistical moments of a stochastic process. The method is based on the derivation of the equations for statistical moment of the response from the FP equation. In general, the statistical moments are governed by an infinite number of coupled equations; a closure technique is used to obtain an approximate solution in terms of a finite set of moments. The accuracy of the solution depends on the order of closure. However, this comes at a price because the method turns out to be impractical for high orders, which are needed for highly nonlinear systems.

3.2.4 STOCHASTIC AVERAGING- The stochastic averaging method was first introduced by Stratonovich in solving nonlinear oscillations of electrical systems under noisy excitations, while a robust mathematical foundation has been established in . In the field of stochastic dynamics most of the works on this topic has been done by Roberts, Spanos and Zhu. Essentially, the method approximates the response vector with a diffusive Markov vector with the probability density function governed by the FP equation. The method is designed to calculate the coefficient function in the FP equation by eliminating the effect of periodic terms by stochastic averaging. The method is applicable to a wide variety of single degree of freedom systems, but it finds its limitation when applied to multi-degree-of-freedom (MDOF) systems.

3.3 Simulations methods

3.3.1 MONTE CARLO SIMULATION - Due to its simplicity, Monte Carlo Simulations is the most frequently applied method to solve random vibration problems. There are no theoretical limitations owing to the nature of the approach; however, for the crude version of MCS, there are computational limitations when the tail of the response distribution is of interest. For highly reliable systems, where the interest is in the far tail of the distribution, many alternative simulation methods have been developed in the recent years. The two principal categories are the IS and MCMC methods.

The importance sampling is a rather straightforward method. The inefficiency of the crude MCS for low probability events lies in the fact that only few samples fall in the failure domain. To avoid this problem, an importance sampling distribution is used in order to generate more samples in the failure domain, making the method more efficient. However, particular care must be taken in using this method in high dimensions, such as in conjunction with discretized stochastic processes. For Gaussian processes in high dimensional spaces, a suitable importance sampling distribution is formulated by Au and Beck. This method is adapted to estimate the first-passage probability of the equivalent linear system obtained by the TELM.

3.3.2MARKOV CHAIN MONTE CARLO METHOD - The Markov Chain Monte Carlo methods are a collection of schemes for sampling from complex probability densities by constructing a Markov chain that has the desired distribution in its equilibrium state . There are different algorithms in this class. The most widely used ones, which can be considered as the parents of all other schemes, are the Metropolis-Hasting algorithm and the Gibbs sampling algorithm. Initially developed outside the field of statistics, these methods greatly impacted statistical analysis in the early 90's, especially in Bayesian computational statistics. In particular, the Metropolis-Hasting algorithm was developed in physics in an attempt to calculate complex integrals as the expected value of random variables by sampling from their distributions. Gibbs sampling found its roots in image processing. Good references for

MCMC methods. MCMC methods are suitable for high-dimensional problems and can be efficiently used to sample in rare failure domains. For this class of problems, the subset simulation method proposed by Au and Beck represents one the most popular simulation method to solve high reliability problems under stochastic excitations.

3.4 Linearization Methods

3.4.1 Equivalent Linearization Method

The equivalent linearization method is the most popular method used in nonlinear stochastic dynamics. Its popularity is based on its simplicity and its wide range of applicability. In particular, its complexity does not increase for MDOF systems and thus it is suitable for civil structures. The general idea behind the method is to replace the nonlinear system by a parameterized equivalent linear system. The method possibly finds its roots in the deterministic linearization method introduced in mechanics by Krylov and Bogolubov. The most appealing feature of every linearization method is that, once the linear system is obtained, all the linear theory can be effortlessly applied.

- Approximates the nonlinear response in terms of an “equivalent” linear system.(Caughey 1963).That equivalent needs to be defined.
- The ELS is determined by minimizing a measure of discrepancy between nonlinear and linear systems. Different methods are characterised by what you are trying to minimize.
- Conventional ELM – minimize the variance of error between nonlinear and linear responses; requires the assumption of a distribution, typically Gaussian (e.g. Atalik and Utku 1976; Wen 1976).Gaussian distribution is used because it simplifies all the calculations. This method works well if you are estimating the variance of the nonlinear response.it provides quite accurate results. However you are interested in **tail probabilities** (the probability that the response will exceed a higher threshold),this method does not works well. this does not work well particularly because of the Gaussian distribution function. we know that even if input is Gaussian, the output of a nonlinear system is not Gaussian.so this Gaussian distribution is rather limited.so the next two methods try to overcome this problem.

- Minimize higher moments of error (Naess 1995) – this method is used for particular kind of elastic nonlinear system where the restoring force has a polynomial form. because he is looking at higher moments, emphasis is placed in the tail and so he is able to get better results in the tail. but the method is restricted again because of the polynomial form.
- Minimizing the difference in mean crossing up rates at a selected threshold(Casciati 1993).by this we can get good results in the tail. however it is not clear how we compute the up crossing rate response

3.4.2 Proposed tail equivalent linearization method

This method defines the linear system by equating the tail probability of the linear response equal to the first order approximation of the tail probability of the nonlinear response (Fujimara and Der Kiureghian 2007).Because it is dealing with the tails the accuracy is enhanced in the tail region.

CHAPTER 4

CHARACTERISTICS OF A LINEAR SYSTEM

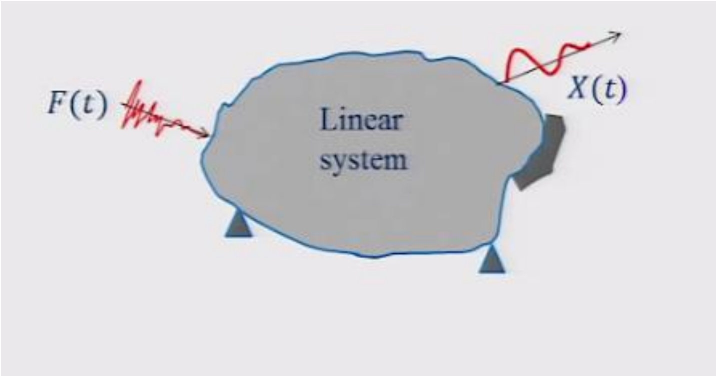

Fig 4.1 Linear System

Consider a linear system.it is subjected to one excitation F(t) and one response X(t).

- For one input-output pair (F(t),X(t)),a stable linear system is completely defined by either of the following:
  - h(t)=impulse response function(IRF),i.e. response to F(t)=δ(t)
  - H(ω)=frequency response function(FRF);i.e. amplitude to steady state response to F(t)=exp(iωt)(complex harmonic function).

If you have either of these functions for a stable linear system, then you have completely characterized the system. You don't need to know the geometry, boundary conditions, etc. So for any input you can contribute the corresponding output.

CHAPTER 5
RELIABILITY

5.1 Reliability Analysis

5.1.1 Reliability:

Reliability is the measure of quality of geotechnical structure over a specified time under standard conditions. In other words reliability is probability of success.

5.1.2 Methods of reliability:

1. First Order Reliability Method (FORM)
2. Second Order Reliability Method (SORM)
3. Monte Carlo Sampling (MCS)
4. Numerical Integration (NI)
5. Increased Variance Sampling (IVS)

Terminology used in reliability:

5.1.3 Mean:

First central moment which is defined as the average value of data set and measures central tendency of data.

5.1.4 Variance:

Second central moment that measures spread in data about mean.

5.1.5 Coefficient of variation (cov):

It measures the dispersion of data. Higher value of cov represents the higher diepersion about its mean.

5.1.5 Covariance:

Covariance indicates the degree of linear relationship between two random variables (x, y).

Cov (x,y) = E ((x-mx) (y-my)) = E (xy- mx my) = E(xy) - E(x)E(y)

The uncertainties in a variable can be quantified using a mathematical model satisfying different functions such as probability density function, probability mass function and cumulative distribution function. Continuous random variable follows normal distribution and beta distribution.

5.1.6 Normal distribution:

The normal distribution is the most widely known and used of all distributions. Because the normal distribution approximates many natural phenomena so well, it has developed into a standard of reference for many probability problems.

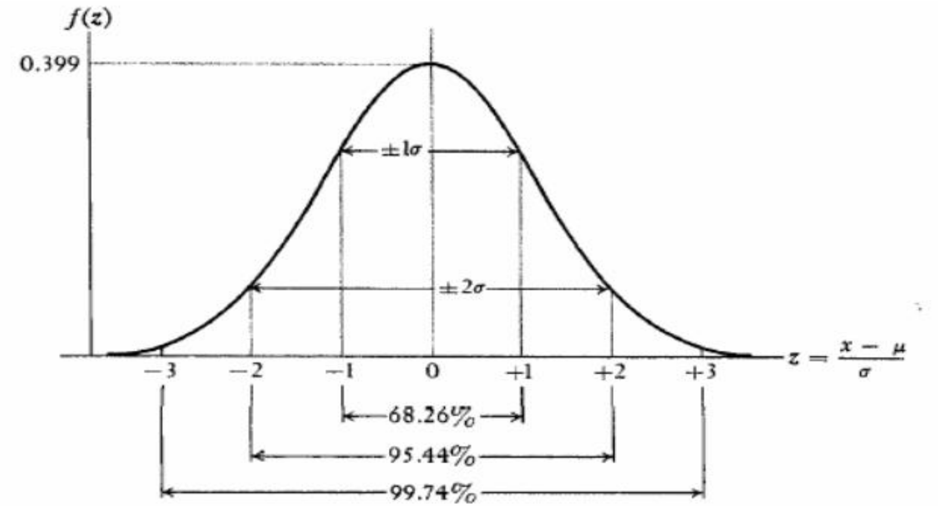

Fig. 5.1 Standard Normal Distribution Curve

5.1.7 Properties of Normal distribution:

- The parameter varies between – ∞ to + ∞ .
- It is perfectly symmetric about mean.
- Mean, Median and Mode values are same.

The rule for a normal density function is

$$f(x;\ \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

Reliability is the probability of success and its value is one minus probability of failure (1-

Pf). If ‘R’ is the resistance and ‘S’ is the load on the structure, then the structure will fail if ‘R’ is less than ‘S’ and probability of failure can be expressed as Pf = P [R≤S] = P [(R-S) ≤0]

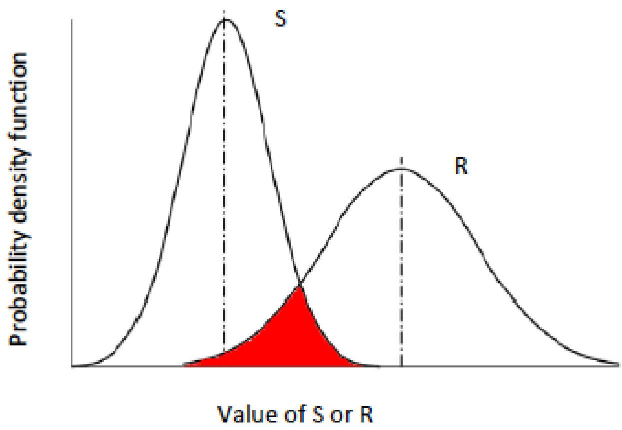

Fig. 5.2 Overlapped area is the probability of failure of random variable R and **Q**

The probability of failure is the shaded area of overlapping as shown in the figure above and mathematically denoted as

$$P_f = \int_{-\infty}^{+\infty} GR(r)Gs(s)ds$$

Reliabilty,

$$R = \int_{-\infty}^{+\infty} GR(r)Gs(s)ds$$

Where GR(r) is CDF of resistance R and Gs(s) is CDF of load S.

Limit state function can be defined as a mathematical model which relates variables such as load and resistance. It is expressed as

Z= (R-S)=f (R,S) = f(*X1,X2,X3, …..,Xn)*

z = margin of safety

If the limit state function is zero then failure would occur and the equation is known as limit state equation. i.e., f(*X1,X2,X3, …..,Xn)* = 0, defines the safe and unsafe which may be linear or non linear.

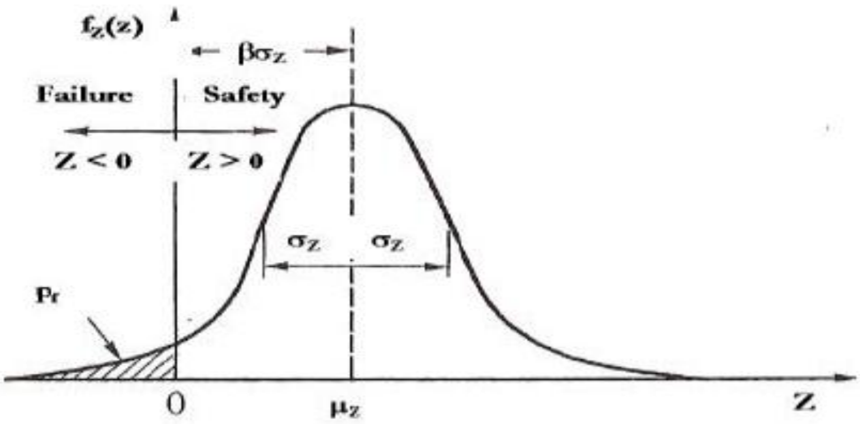

Fig. 5.3 Distribution of safety margin (Melchers 2002**)**

Cornell gave expression for reliability index

$\beta = \frac{\mu z}{\sigma z}$ $and\ P_f = \phi(-\beta)$ is CDF of standard normal variable.

5.2 First order reliability method (FORM)

It is a well developed method for structural reliability analysis.

- An approximate method for solving time-invariant reliability problems.
- X=vector of random variables.

g(x)=limit state function (g(x)<=0→failure event)

p(f)=Pr[g(x)<=0] probability of failure

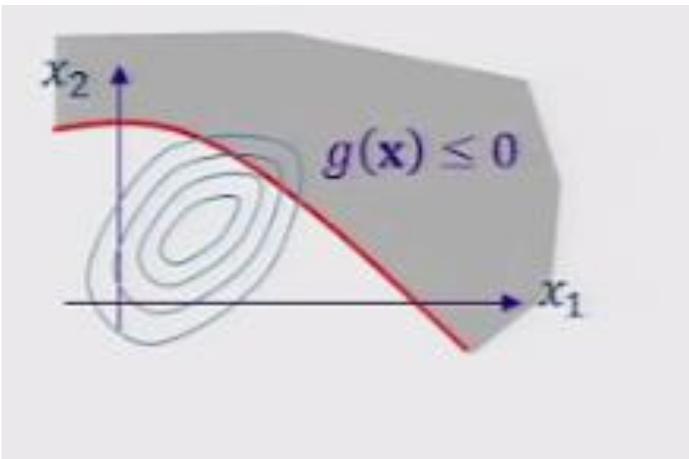

Fig 5.4 Geometry of random variables

The picture above describes the concept in terms of geometry in the space of this random variables x1 and x2.The contours are representing the contour of the probability density function of these random variables. The red line is the limit state surface where this limit state function takes zero values and the grey domain is the failure domain. The task is to compute the probability of the random variables to be in this failure region.

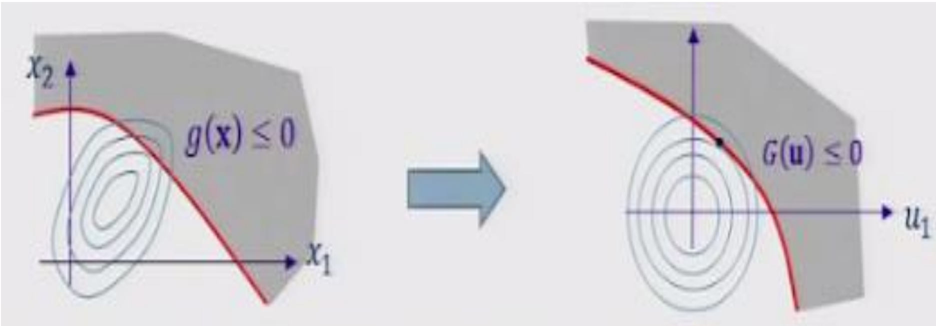

Fig 5.5 Transformation from x space to u space

The FORM solves this problem by making a transformation.We make a transformation from the x space to u space (a vector of standard normal variables).There is no approximation involved here and this can be done as long as the random variables are continuous and have a strictly increasing joint cumulative distribution function (cdf).

- u=u(x) transformation to standard normal space.
- G(u)=g{x(u) limit state function in transformed space.

The advantage of doing this(x→u space) is that in u space the probability densities have contours that are spherical and hyper spherical in higher dimensions. So it is an canonical space and in this space there are simple properties in terms of probability computations.
Next we find the point nearest to the origin and we call this the **design point.**

u*=minarg[(||u|| / G(u))=0] design point

We linearize the surface at that point.
$\beta = ||u^*||$ reliability index is the distance from the origin to design point and in standard normal space the mean is at zero so the farther you are from the mean, the farther the failure domain is from the mean point, the more reliable it is.so this distance is the measure of the reliability.

The first order approximation of the failure probability is described by the probability of failure described by that hyperplane which would be half space probability in the standard normal space that depends only on the distance of the origin due to rotational symmetry.It is the standard normal probability function evaluated at minus the distance from the origin.

$$P_f=\Phi(-\beta)\text{, FORM approximation}$$

Fig 5.6 Design point and reliability index representation

This works well because of the fact that in standard normal space probability density decays exponentially with the distance from the origin, so as we go far away from the origin, the discrepancies between the actual surface and the hyperplane become negligible.

CHAPTER 6

DISCRETE REPRESENTATION OF STOCHASTIC PROCESS

To use a time-invariant stochastic problem we have to discretize that the stochastic process can be only represented in terms of random variables.

6.1 General form of a zero-mean Gaussian process

F(t) = s(t).u

S(t) = [s(t1)……..sn(t)] vector of deterministic basis function that carry time evaluation of a process

U=[u1…..un] vector of standard normal variables that brings in the stochasticity.

This is a way of separating variation in time and stochasticity.

There are different ways of doing this.

6.2 Time domain discretization (modulated filtered white noise)

S1(t)=q(t)hf(t-t1) hf(.) = impulse response function of a linear filter.

Q = modulating function that modulates the process in time.

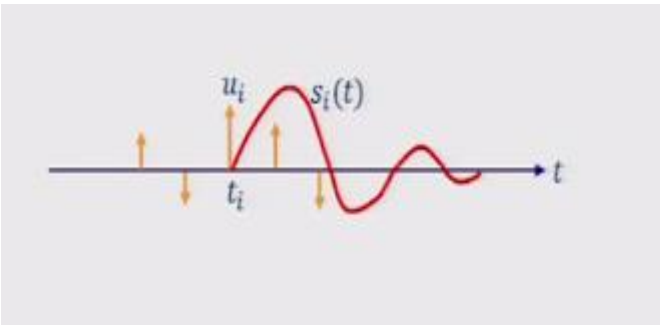

Fig 6.1 Time domain discretization

The picture above shows what it means. we have discretized time.At each time we have a random impulse and the filter responses to that pulse and you can sum up because the first equation is nothing but a summation.so when you sum up you end up getting as in picture shown below

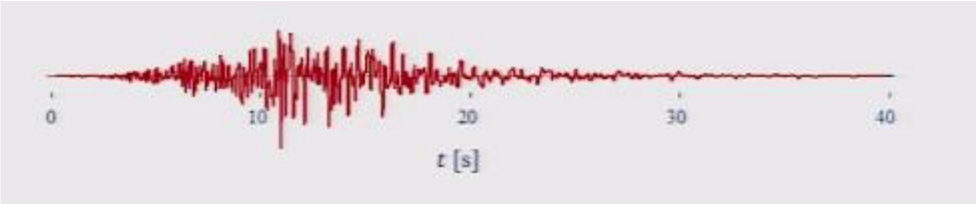

Fig 6.2 Frequency domain discretization

This process is not only non-stationary in time but also non stationary in frequency domain.

6.3 Frequency-domain discretization (stationary process)

$$F(t) = \sum_{i=1}^{n/2} \sigma_i [u_i \sin(\omega_i\, t) + u_i \cos(\, \omega_i t)]$$

$$s_i(t) = \sigma_i \sin(\omega_i t), s_i(t) = \sigma_i \cos(\omega_i)$$

This is a very well known way of decomposing the process into its frequency components

CHAPTER 7

FORM SOLUTION OF STOCHASTIC DYNAMIC PROBLEMS

How to use FORM to solve stochastic dynamic problems

7.1 Definitions

- F(t)=s(t).u discretized stochastic excitation
- X(t,u)=response to discretized stochastic excitation(the response now is a function of time but also implicitly the function of this random variables u and there could be many of them depending on how you discretize duration and so on.
- Pr(x<X(t,u))=tail probability at threshold x at time t.(the tail probability is the probability that at a given time t, the response exceeds a threshold x.

7.2 Reliability Formulation

- G (u,x) = x-X(t,u) → here is the limit state function. G because the random variables are already in the space.
- Pr (x<X(t,u) = Pr(G(u,x)≤0) →tail probability becomes the probability that the limit state function takes the negative value.
- u*=argmin(||u|| / G(u,x)=0) →design point
- $\beta(x)=\|u^*(x)\|$

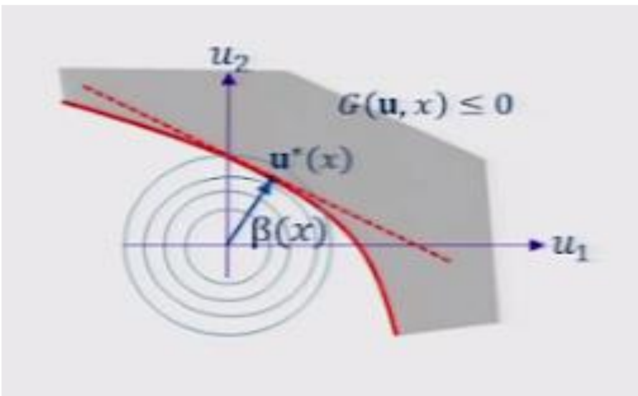

Fig 7.1 Reliability index (β(x))

This is similar to shown before with the exception that the limit state function has the threshold as a parameter also.

- Pr(x<X(t,u)) → Φ(-β(x)) FORM approximation of tail probability.

This distribution is not Gaussian because β is not necessarily proportion to x.

7.2.1 Realibility formulation- in case of linear system

We can use superposition principle.

- X(t,u) = a(t).u $a_i(t)$=collection of responses to deterministic functions $s_i(t)$.The response is a linear function of u.
- G(u,x) = x-a(t).u → the limit state function is a linear function of u
- $u^* \ (x) = \frac{x(a(t))}{(||a(t)||^2)}$
- $\beta(x) \ = \frac{x}{||a(t)||}$ → reliabilty index is proportional to threshold.
- Pr (x<X(t,u)) = Φ(-β(x)) → tail probability β is proportional to x, so this shows that the response is Gaussian.

CHAPTER 8
IDENTIFICATION OF THE LINEAR SYSTEM

Given the design point, one can identify the linear system (for the input-output pair) - if we are given u*,we can find the linear vector a and once we have vector a, we can identify the system either in time domain or in frequency domain.

u* → a(t)

8.1 Time domain analysis:

Solve for h(t) in system of equations

$$\sum_{j=1}^{n} h(t-t_j)s_i(t_j)\Delta t \;=\; a_i(t),\;\; i=1,\ldots\ldots,n$$

The vector a_i are responses to the deterministic functions s_i. This is the discretized version of the Duhamal's integral.

If we know a_i and s_i we can compute h at different time steps.so we can obtain the unit impulse response function if we have the design point.so even if we don't know the linear system. By knowing design point we can know which type of linear system we are using.

8.2 Frequency domain analysis :

$$|H(\omega_i| = \frac{\sqrt{a_i(t)^2+\bar{a}_i(t)^2}}{\sigma_i}$$

$$\theta_i = \tan^{-1}\,[\frac{a_i(t)}{\bar{a}_i(t)}]$$

$$H(\omega_i) = |H(\omega_i)|\exp\,(i\theta_i)$$

Given the a_i we can compute the modulus and the phase angle of the frequency response function.

CHAPTER 9
THE TAIL EQUIVALENT LINEARIZATION METHOD

9.1 Introduction

TELM is a new linearization method for nonlinear stochastic dynamic analysis introduced by Fujimura and Der Kiureghian.

It makes use of the time- invariant first-order reliability method (FORM) to accurately estimate the tail of the distribution of the response of a nonlinear system that is subjected to a stochastic input.

In TELM the input process is discretized and represented by a set of standard normal random variables. Each response threshold defines a limit state surface in the space of these variables with the "design point" being the point on the surface that is nearest to the origin. Linearization of the limit-state surface at this point uniquely and non- parametrically defines a linear system, denoted as the tail-equivalent linear system, TELS. The tail probability of the response of the TELS for the specified threshold is equal to the first-order approximation of the tail probability of the nonlinear system response for the same threshold.

Once the TELS is defined for a specific response threshold of the nonlinear system, methods of linear random vibration analysis are used to compute various response statistics of interest, such as the mean crossing rate and the tail probabilities of local and extreme peaks.

The method has been developed for application in both time and frequency domains and it has been applied for inelastic structures as well as structures experiencing geometric nonlinearities.

9.2 Steps in TELM

- For selected threshold x and time t, formulate tail probability problem in terms of limit state function

$$G(u,t) = x - X(t,u)$$

- Find the design point u*
- Find the gradient vector of the tangent plane $a(t) = \frac{xu^*}{||u^*||^2}$
- Identify the tail-equivalent linear system TELS that corresponds to gradient vector a in terms of its IRF h(t) or its FRF H(ω)→TELS is defined by the tangent at that

hyperplane. The computation is straightforward. The most difficult thing is finding the design point.

9.3 Iterative algorithms for solving design point

$u^*(x) = \arg\left\{\frac{||u||}{G(u.,x)} = 0\right\}$ requires repeated computations of X(t, u) and gradient of response for selected values of u.

The design point is the solution to constraint optimization problem. We want to minimize the distance from origin to limit state surface.
So to find the design point we have to repetitively solve the nonlinear problem. Not many times ,we typically converge in 10-20 steps.
To find the gradient of response, if we use finite difference method, it can be very costly because number of random variables used is large.
So we make use of direct differentiation method.

For many thresholds→

For an ordered sequence x1<x2<…..<xn.

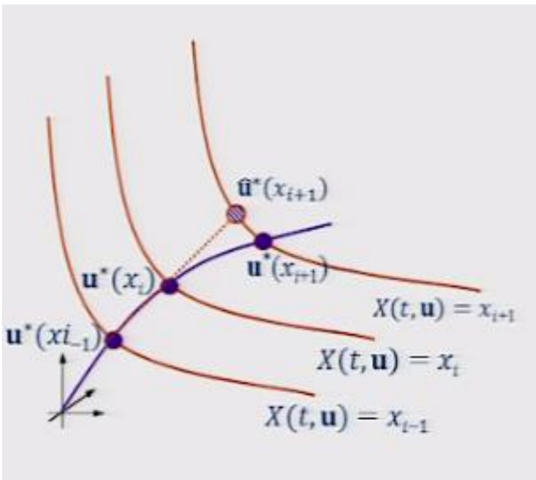

Fig 9.1 Representation of design point

First we find out two points on the trajectory, then extrapolate to get the remaining points.

$$\hat{u}^* \ (x_{i+1}) = u^* \ (x_i) + \lambda \ \frac{u^*(x_i) - u^*(x_{i-1})}{||u^*(x_i) - u^*(x_{i-1}||}$$

CHAPTER 10

CHARACTERICTICS OF THE TAIL EQUIVALENT LINEARIZATION METHOD

- **For a given threshold x and time t,**

The tail probability of the TELS response = first order approximation of the tail probability of the nonlinear system response. Hence the name Tail Equivalent Linearization Method (TELM).

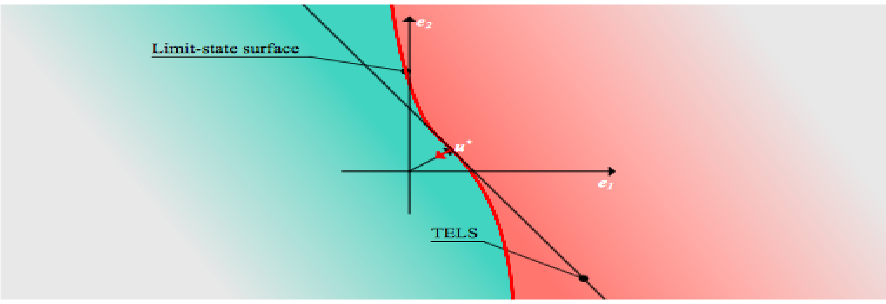

Fig 10.1 TELS of the non linear response for a given threshold x and point in time t

- As opposed to ELM and other linearization methods, TELM is a **Non-parametric method**. The conventional equivalent linearization method is a parametric method i.e. you have to define a parameterised linear system and through optimization we can find the parameters of the linear system.
- There is no need to define a parameterized linear system. The tail-equivalent linear system, TELS, is introduced and numerically identified in terms of its IRF and/or FRF for a specific response threshold. A one-to-one relationship exists between the design point of the tail distribution and the IRF/FRF of a linear system. In particular, the coordinates of the design point are sufficient to determine the IRF/FRF. In the nonlinear case, this one-to-one relationship completely characterizes the TELS when linearization is employed at the design point of the nonlinear system. Remarkably, TELS is a non-parametric linear system in the

sense that no parameterized model needs to be defined. Even the order of the system need not be determined

- The design point excitation $F^*(t)= s(t).u^*$ represents the **most likely realization** of the stochastic excitation to give rise to the event $\{x \leq X(t,u)\}$ → once we find the design point u^* and we put it back in the expression of discretized point excitation, we find the design point excitation. The meaning of this is that it is the most likely realization of the excitation process to give rise to the event of interest.

10.1 Numerical Example

In this section the properties of TELM are numerically investigated by considering a single degree of freedom (SDOF) oscillator with inelastic material behaviour. The problem is solved both in frequency and time domains. We use a symmetric Bouc-Wen material model to describe the force-displacement relationship. Other inelastic material models can be used in the formulation.

However, there is a fundamental condition for application of TELM: the limit-state function and therefore, the response of the system must be differentiable with respect to the random variables u at the design point. This guarantees that the limit-state surface has a tangent hyper plane at the design point. It has been proven in that, for an inelastic material, a necessary condition for the differentiability of the response is a smooth transition between material states except for elastic unloading. This condition is satisfied for the Bouc-Wen material model, but in other cases the material model may have to be modified to have smooth transitions.

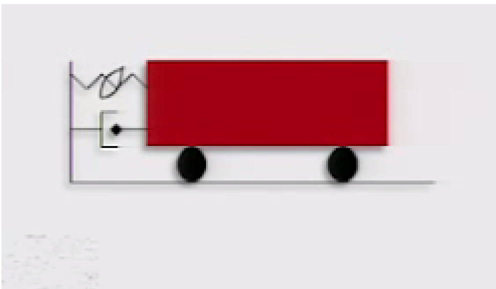

Fig. 10.2 SDOF oscillator with inelastic material behaviour

To gain insight into the nature of the TELS, consider a hysteretic oscillator defined by the differential equation

$$m\ddot{X}(t) + c\dot{X} + k[\ \alpha X(t) + (1-\alpha)Z(t)] \ = \ F(t)$$

where m = 3.0x10^5 (kg) , c=1.5x10^2 (kN s/m) and k=2.1x10^4 (kN/m) are assumed.

The parameter α, which controls the degree of hysteresis, is set to α = 0.1.

The excitation is defined as $F(t) = -m\ddot{U}g(t)$,where $\ddot{U}$g(t) denotes the base acceleration modelled as a white-noise process.

The TELS is independent of the scale of the excitation. Therefore, any finite value for the intensity of the white noise produces the results given below. The term $Z(t)$ in follows the Bouc–Wen hysteresis law

$$\dot{Z}(t) = -\gamma|\dot{X}||Z(t)|^{n-1}\ Z(t) - \ \eta|Z(t)|^{n}\dot{X}(t) + A\dot{X(t)},$$

where the $\gamma = \eta = 1/2\sigma o^{n}$ and parameters are selected as n=3,A=1, and $\sigma_o{}^2$=$\frac{\pi S m^2}{ck}$ is the mean square responseof the linear ($\alpha = 1$) oscillator

We can change the values of stiffness, mass,damping ratio, initial displacement, velocity,natural period and the graph given above will change correspondingly.

The following example which investigates the properties of TELM is solved on MATLAB. The various parameters whose values are given is used in the code and some values of other parameters have been assumed.Some predefined function such as “pwelch,linsquare,hilbert,linsquare” have been used which are already defined in MATLAB. We plot the graphs between ground acceleration vs time,show the variation of impulse response functions(IRFs) and frequency response functions(FRFs) to show the exact nature of TELM.The problem has been solve both in time and frequency domain.

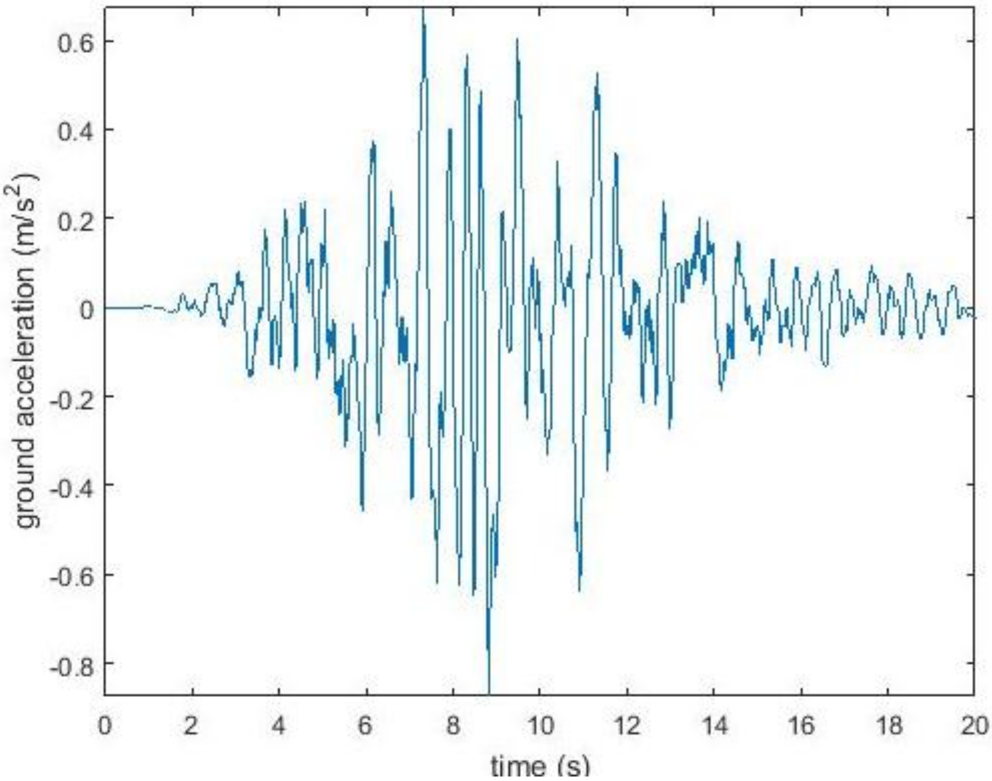

Fig.10.3 ground acceleration vs time graph

The above figure shows that the ground acceleration $\ddot{U}$g(t) reaches a peak and then its effect diminishes after some time.

- The TELS is independent of a scaling of the excitation since the direction of the design point or the shape of the limit-state surface is invariant of this scaling. i.e. h(t,x) and H(ω,x) for excitation sF(t) are invariant of s.

This characteristic is central in obtaining fragility curves. Fragility is conditional probability of event of interest conditioned on scale of excitation.

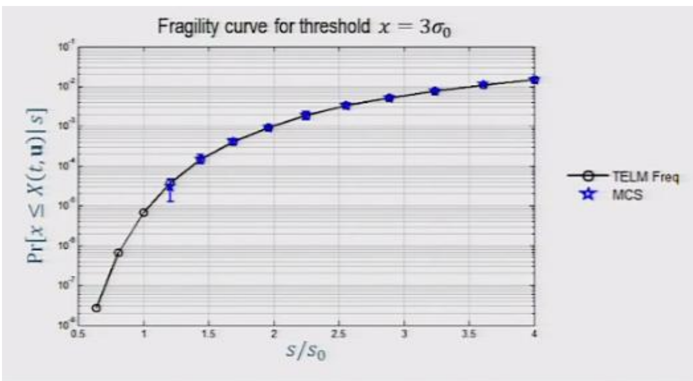

Fig.10.3 Fragility curve for given threshold

All this curve has been developed from one design point.

- For broad-band excitations, the TELS is mildly dependent on the frequency content of the excitation. Hence, a white-noise approximation can be used to determine the IRF/FRF. For narrow-band excitations, this is no longer valid and the IRF/FRF must be determined for the specific input power spectral density.

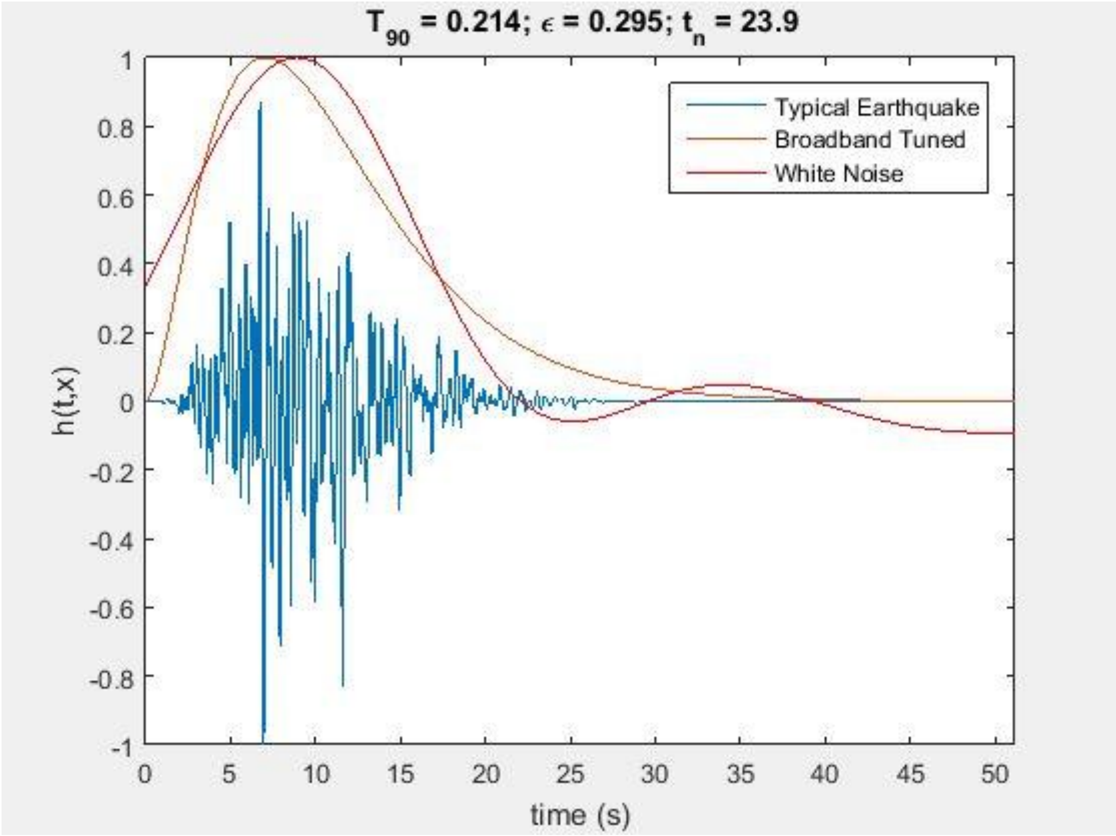

Fig.10.4 IRFs of TELS for hysteretic oscillator response to white noise

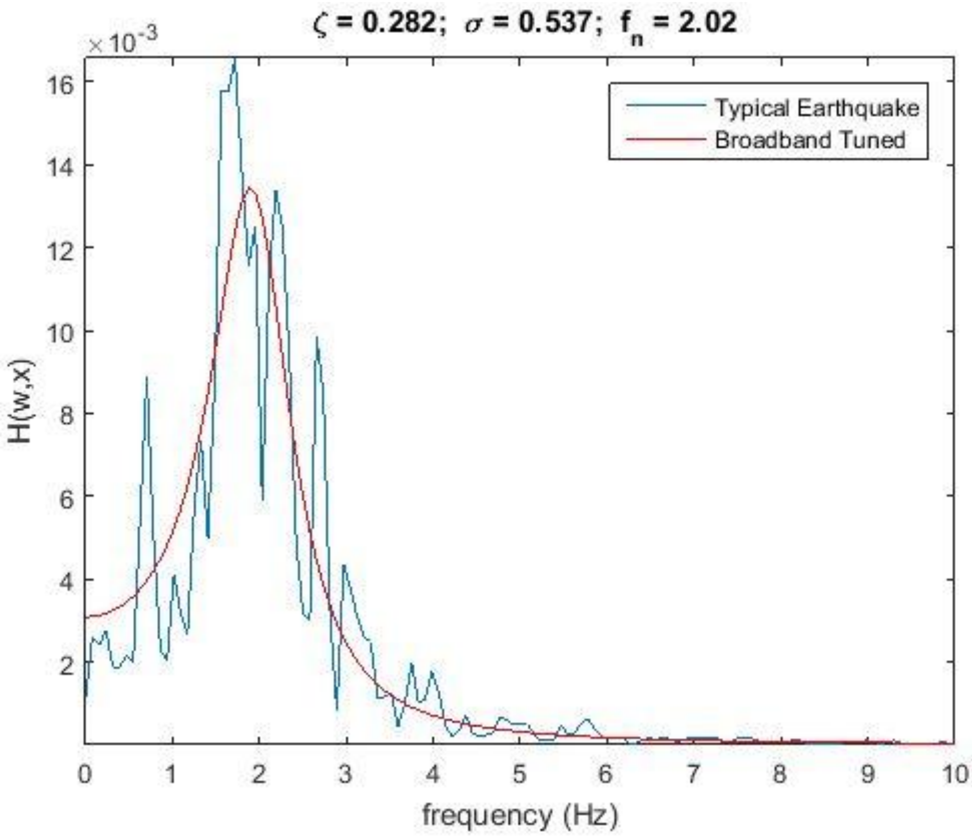

Fig.10.5 FRFs of TELS for hysteretic oscillator response to white noise

- Influence of non-stationarity on TELS.

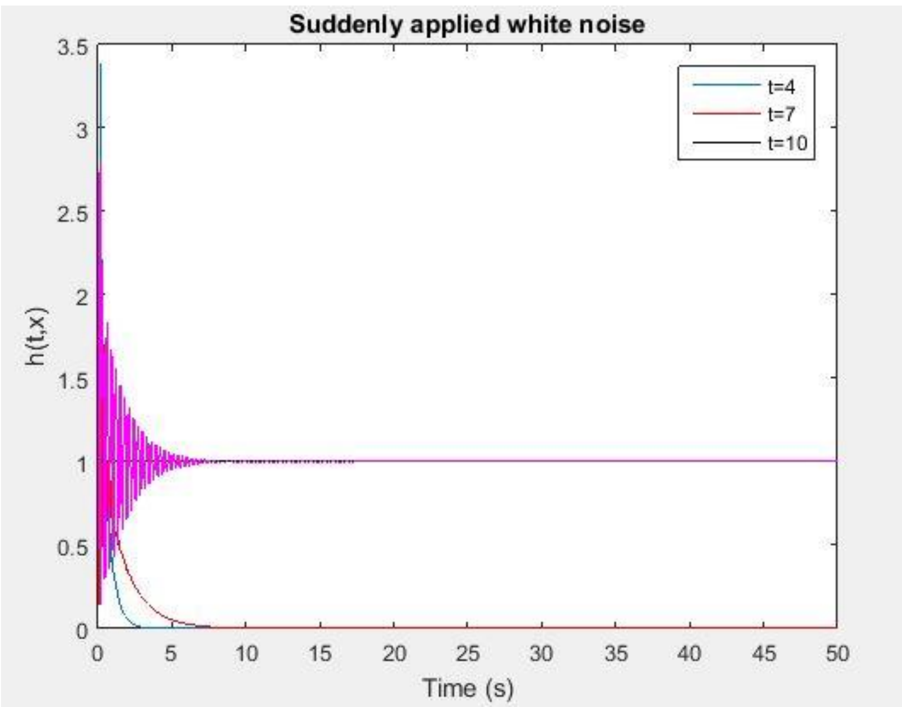

Fig.10.6 Influence of non-stationarity on the IRFs of the TELSs

The above figure compares the corresponding IRFs for t_n=4s,7sand 10s,which are plotted for interval (0,5s). For the case of a suddenly applied stationary excitation, little dependence of IRF on t_n is evident. It follows that, whereas for a stationary process a single IRF per threshold is adequate, for a non-stationary process one may need to determine the IRF at each time point where the response statistics are required. This is similar to the ELM, where for non-stationary excitation the equivalent linear system must be determined at each time step.

- The TELS strongly depends upon the selected thresholds:

  $h(t) \rightarrow h(t,x)$, $H(\omega) \rightarrow H(\omega,x)$

In conventional linear system there is one linear equivalent linear system that you find and you have to apply for all thresholds but here for each linear system you end up having different linear system and for that reason we designate the unit impulse response function and frequency response function of the equivalent linear system as functions of threshold x also.

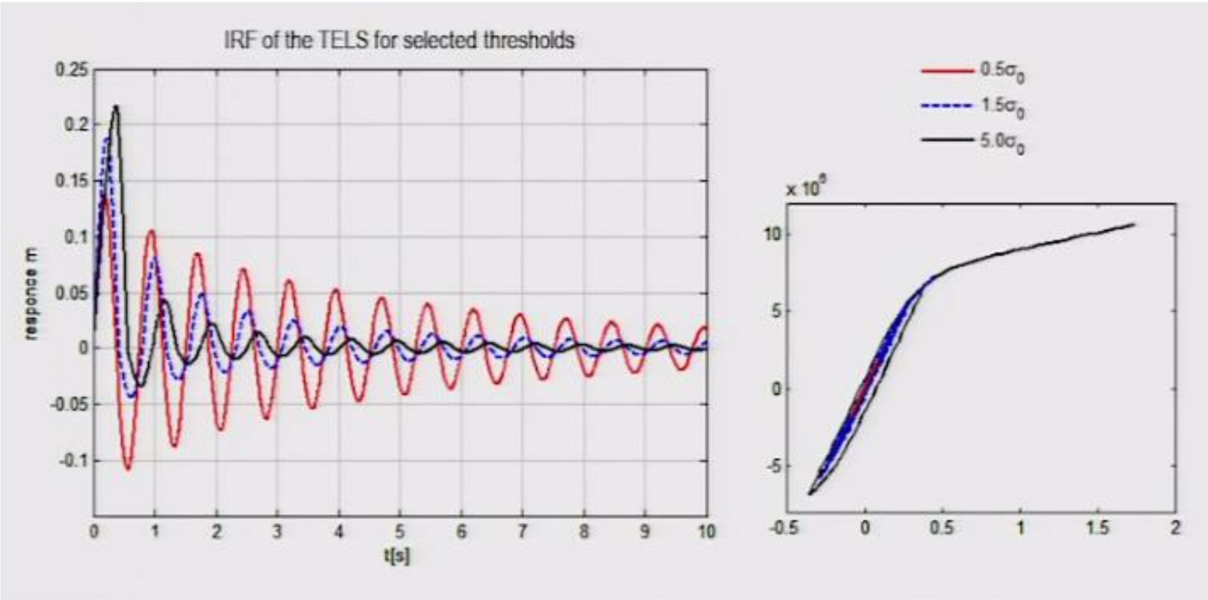

Fig.10.7 variation of IRF of the TELS for selected thresholds

As the threshold x increases, the dissipation is faster. These curves are not typical of the unit response function of the linear oscillator.

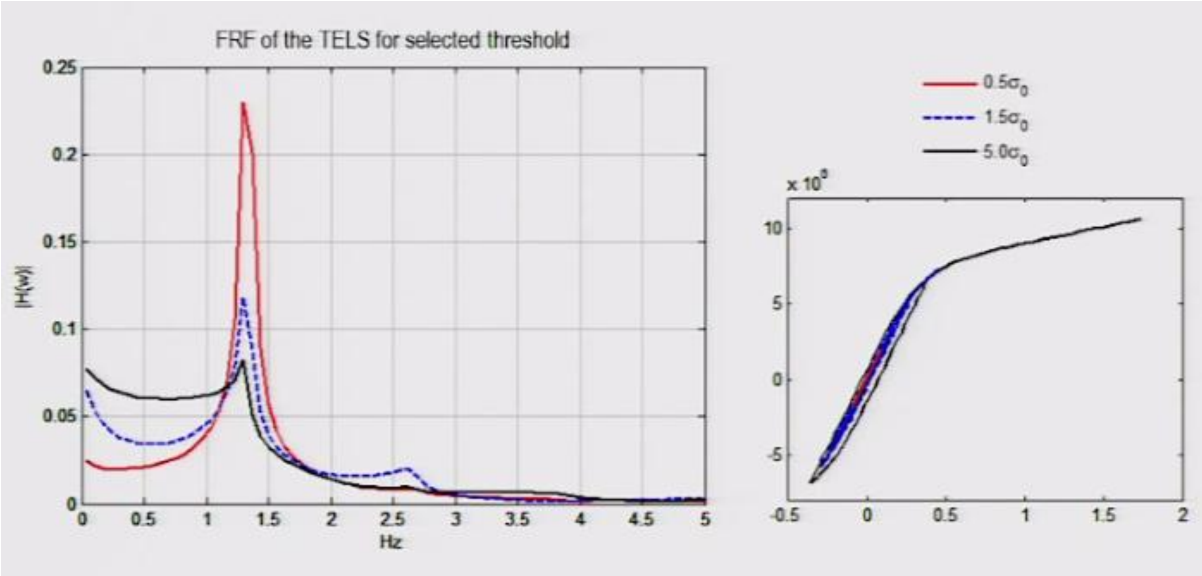

Fig.10.8 variation of FRF of the TELS of selected threshold

As the threshold increases the peak drops.

- Because of the dependence of the TELS on the threshold, TELM is unable to capture the non-gaussian distribution of the nonlinear response.

$Pr[x \leq X(t,u)] = \Phi(-\beta(x))$

Because the reliability index is not proportional to x, the TELM is able to capture the non-gaussian distribution of the nonlinear response.

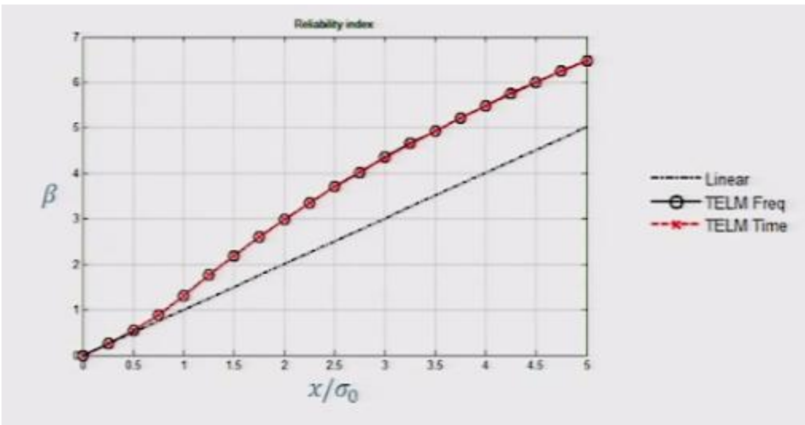

Fig.10.9 Variation of reliability index with threshold

Blue-linear(α=1) Red-nonlinear(α=0.1)

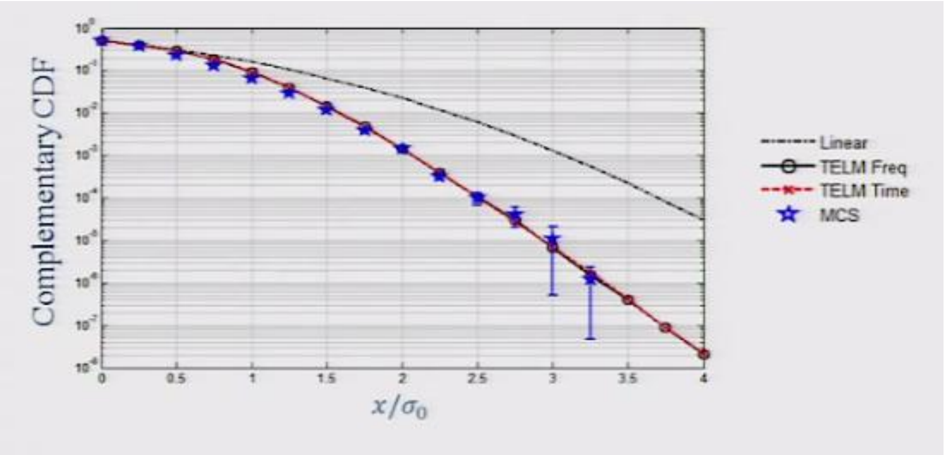

Fig.10.10 Variation of complementary cdf with threshold

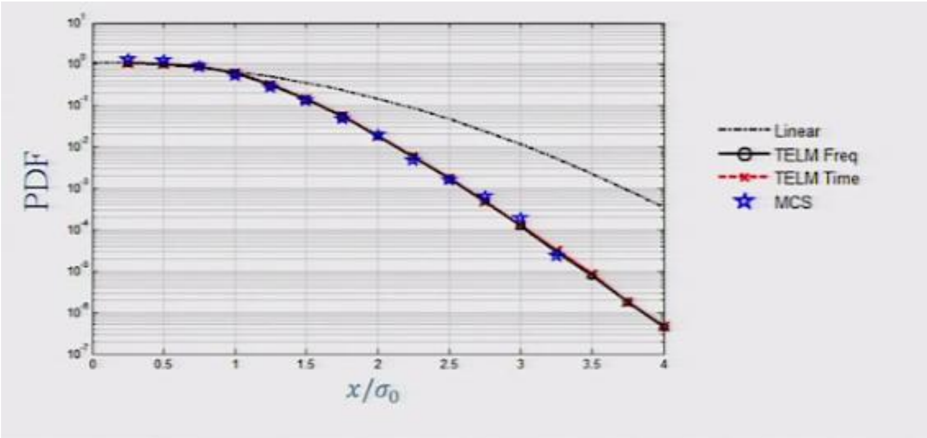

Fig.10.11 Variation of probability density function with threshold.

If the response is Gaussian, the curve would appear to be parabolic. For linear response the curve is parabolic. The curve shown above is not parabola. The tail goes down as a straight line.

- For stationary response TELS is invariant of time t. Thus TELSs determined for one time point are sufficient to evaluate all statistical properties of the response e.g.,
  - **Point-in-time distribution** Pr[x≤X(t,u)]
  - **Mean up crossing rate**
  - **First passage probability** i.e., the probability distribution of the maximum response over an interval of time is presented. You do not want point in time, you want the distribution throughout.
- TELM is easily extended to MDoF systems-number of random variables remain the same.

CHAPTER 11

LIMITATIONS AND SHORTCOMINGS OF TELS

- The classical drawbacks of FORM also apply to TELM. In particular, there is no measure of the error due to the linearization approximation, which means that the accuracy of TELM cannot be estimated in advance. Moreover, TELM requires far more analysis than ELM, if one is interested only in the first and second moments of the nonlinear response. Thus, for second-moment analysis, ELM is the appropriate method, while TELM is the appropriate method to use for estimation of tail probabilities.
- TELM requires many repeated computations. We use Direct Differentiation method for this purpose.
- The nonlinear response must be continuously differentiable-must use smooth or smoothened constitutive laws otherwise you cannot define the tangent plane. we cannot use purely elasto-plastic oscillator. we have to smoothen that elasto-plastic oscillator. the transition between different systems have to be smooth.
- The limit state surface must be well behaving. TELM does not works well for strongly stiffening systems(e.g. Duffin oscillator with a strong cubic term) or when non linearity involves abrupt behaviour in the system behaviour.
- TELM is not applicable to degrading systems.

CHAPTER 12

CONCLUSIONS

- TELM is an alternative linearization method for nonlinear stochastic dynamic analysis.
- TELM is non parametric
- Captures non-gaussian distribution of nonlinear response.
- Offers superior accuracy for tail probabilities.
- Is particularly convenient for fragility analysis.
- Can be applied to stationary and non-stationary response.
- Can be applied to MDoF systems, multi-component excitations.
- TELM requires continuous differentiability of the nonlinear response.
- As other linearization methods, the accuracy of TELM depends on the nature of the nonlinearity.

APPENDIX-I

Matlab code for numerical example solved in chapter 10:

1.Main.m-

```
%% Tail-equivalent linearization method for nonlinear random vibration
clear all;close all;clc; warning off;
f = linspace(0,40,2048); % frequency vector
zeta = 0.3; % bandwidth of the earthquake excitation.
sigma = 1; % standard deviation of the excitation.
fn =2; % dominant frequency of the earthquake excitation (Hz).
T90 = 0.3; % value of the envelop function
eps = 0.4; % normalized duration time
tn = 20; % duration of ground motion (seconds).
f0   = 0;
Fs   = 100;  % (in Hz) Frequency sample
NFFT = 2^12; % number of frequencies for discretization of IFT (4096)

% function call
[y,t] = seiTELS(sigma,fn,zeta,f,T90,eps,tn);
% y: acceleration record
% t: time

 input.Vs          = [2 3 2]     % (m/s)
input.rho         = [2000 2100 2400];    % psd (kgr/m3)
input.damp        = [0.04 0.03 0.01];    % damping ratio
input.freq        = linspace(f0,Fs,NFFT);% frequency range
input.layer_thick = [10 10];            % (m) ! no thickness for bedrock!

% Call function
[f, U, A, B] = HOR_IRF(input);
```

```
% Frequency response function
FRF_linear = U(1,:)./U(end,:);
FRF_firstorder = U(2,:)./U(end,:);
FRF_secondorder = U(3,:)./U(end,:);
FRF_thirddorder = U(2,:)./U(end,:)+1;
%% plot
figure
plot(t,y);
xlabel('time (s)')
ylabel('ground acceleration (m/s^2)')
axis tight ;xlim([0 20]);
set(gcf,'color','w')
%% Fitting the ground acceleratoin record to target spectra & envelop
guessEnvelop=[0.33,0.43,50]; % guest for envelop
guessKT = [1,1,5]; % guess for spectrum
[T90,eps,tn,zeta,sigma,fn] = KTPSD(t,y,guessEnvelop,guessKT,...
    'dataPlot','yes')
% plot
figure;
plot(f,abs(FRF_linear));
hold on;
plot(f,abs(FRF_firstorder),'r');
hold on;
plot(f,abs(FRF_secondorder),'k');
hold on;
plot(f,abs(FRF_thirddorder),'m');
xlim([0 Fs/2])
xlabel('Time (s)')
ylabel('h(t,x)')
title('Suddenly applied white noise');
legend('t=4','t=7','t=10');
```

2.seiTELS.m-

```
function [y,t] = seiTELS(sigma,fn,zeta,f,T90,eps,tn)
% [y,t] = seiTELS(sigma,fn,zeta,f,T90,eps,tn) generate one time series
% corresponding to acceleration record from a seismometer. The function
% requires 7 inputs, and gives 2 outputs. The time series is generated in
% two steps: First a stationnary process is created based on the Kanai-
% Tajimi spectrum, then an envelope function is used to transform this
% stationnary time series into a non-stationary record.

%% Initialisation
w = 2*pi*f;
fs = f(end);
dt = 1/fs;
f0= median(diff(f));
Nfreq = numel(f);
t = 0:dt:dt*(Nfreq-1);

%% Generation of the spectrum S
fn = fn*2*pi; % transformation in rad;
s0 = 2*zeta*sigma.^2./(pi.*fn.*(4*zeta.^2+1));
A = fn.^4+(2*zeta*fn*w).^2;
B = (fn.^2-w.^2).^2+(2*zeta*fn.*w).^2;
S = s0.*A./B; % single sided PSD


%% Time series generation - Monte Carlo simulation
A = sqrt(2.*S.*f0);
B =cos(w'*t + 2*pi.*repmat(rand(Nfreq,1),[1,Nfreq]));
x = A*B; % stationary process

%% Envelop function E
b = -eps.*log(T90)./(1+eps.*(log(T90)-1));
```

```
c = b./eps;
a = (exp(1)./eps).^b;
E = a.*(t./tn).^b.*exp(-c.*t./tn);

%% Envelop multiplied with stationary process to get y
y = x.*E;

end
```

3.HOR_IRF.m-

```
function [f, U, A, B] = HOR_IRF(input)


if length(input.Vs) ~= length(input.layer_thick)+1
    disp('There is a problem with the number of velocities Vs assigned to the various layers')
    disp(' ')
    disp('Solution: Assign velocities for the all soil layers and for the bedrock')
end

% frequency vector ((f0 Fs NFFT)=(0 100 4096) 0 to 100 in 4096 parts)
f = input.freq;

% circular frequency vector again angular frequency vector
omega = 2*pi*input.freq;

% imaginary "i" i is defined here
clear i; i=sqrt(-1);

% complex shear wave velocity input.Vs=[2 3 2]
Vsstar = input.Vs.*(1+i*input.damp);

% thickness of the soil layers
h = input.layer_thick;
```

```
% number of soil layers + bedrock, it shows size of vector here it is 3
layernum = length(input.Vs);

% complex impendance ratio on layer interfaces
az=zeros(layernum-1); % 0 matrix of 2x2
for i1 = 1:layernum-1 % it shows loop will run for layernum - 1 times that is 2
   az(i1) = input.rho(i1) * Vsstar(i1) / ( input.rho(i1+1) * Vsstar(i1+1) );
end

% Initialization of matrices
kstar = zeros(layernum,length(input.freq));% matrix size 3x4096
A     = zeros(layernum,length(input.freq));
B     = zeros(layernum,length(input.freq));
U     = zeros(layernum,length(input.freq));

% Calculate transfer functions
for i1 = 1:layernum  % Loop for the soil layers

   for i2 = 1:length(input.freq)  % Loop for the frequencies

      kstar(i1,i2) = omega(i2)./Vsstar(i1);  % complex wave number kstar=omega/Vsstar

      if i1 == 1
         A(i1,i2) = 0.5*exp(i*kstar(i1,i2)*input.layer_thick(i1)) + ...
            0.5*exp(-i*kstar(i1,i2)*input.layer_thick(i1));

         B(i1,i2) = 0.5*exp(i*kstar(i1,i2)*input.layer_thick(i1)) + ...
            0.5*exp(-i*kstar(i1,i2)*input.layer_thick(i1));

         U(i1,i2) = A(i1,i2) + B(i1,i2);
      else
         A(i1,i2) = 0.5*A(i1-1,i2) * (1+az(i1-1)) * exp(i*kstar(i1-1,i2)*input.layer_thick(i1-
1)) + ...
```

```
          0.5*B(i1-1,i2) * (1-az(i1-1)) * exp(-i*kstar(i1-1,i2)*input.layer_thick(i1-1));

        B(i1,i2) = 0.5*A(i1-1,i2) * (1-az(i1-1)) * exp(i*kstar(i1-1,i2)*input.layer_thick(i1-1))
+ ...
          0.5*B(i1-1,i2) * (1+az(i1-1)) * exp(-i*kstar(i1-1,i2)*input.layer_thick(i1-1));

        U(i1,i2) = A(i1,i2) + B(i1,i2) ;
      end %end if

    end %end i2

end %end i1

N = length(input.freq);

% Complex conjugates for "perfect" ifft
if round(rem(N,2))==1
    ia = 2:1:(N+1)/2;
    ib = N:-1:(N+3)/2;
else ia = 2:1:N/2; ib = N:-1:N/2+2;
end

A(:,ib) = conj(A(:,ia)) ;
B(:,ib) = conj(B(:,ia)) ;
U(:,ib) = conj(U(:,ia)) ;
```

4. KTPSD.m-

```
function [T90,eps,tn,zeta,sigma,fn] = KTPSD(t,y,guessEnvelop,guessKT,varargin)
%
%Kanai–Tajimi model psd to ground acceleration
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% INPUTS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% y: size: [ 1 x N ] : aceleration record
% t: size: [ 1 x N ] : time vector
% guessEnvelop: [1 x 3 ]: first guess for envelop function
% guessKT: [1 x 3 ]: first guess for Kanai–Tajimi spectrum
% varargin:
%     'F3DB'       - cut off frequency for the low pass filter
%     'TolFun'     - Termination tolerance on the residual sum of squares.
%                  Defaults to 1e-8.
%     'TolX'       - Termination tolerance on the estimated coefficients
%                  BETA.  Defaults to 1e-8.
%     'dataPlot'      - 'yes': show the results of the fitting process
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% OUTPUTS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% sigma: [1 x 1 ]: Fitted standard deviation of the excitation.
% fn: [1 x 1 ]:  Fitted dominant frequency of the earthquake excitation (Hz).
% zeta: [1 x 1 ]: Fitted bandwidth of the earthquake excitation.
% f: [ 1 x M ]: Fitted frequency vector for the Kanai-tajimi spectrum.
% T90: [1 x 1 ]: Fitted value at 90 percent of the duration.
% eps: [1 x 1 ]: Fitted normalized duration time when ground motion achieves peak.
% tn: [1 x 1 ]: Fitted duration of ground motion.

%% inputParser
whiteNoise = inputParser();
whiteNoise.CaseSensitive = false;
whiteNoise.addOptional('f3DB',0.05);%3 decibel frequency
whiteNoise.addOptional('tolX',1e-8);% Adds an optional argument to the input scheme. tolerance 10^(-8)
whiteNoise.addOptional('tolFun',1e-8);
```

```
whiteNoise.addOptional('dataPlot','no');
whiteNoise.parse(varargin{:});
tolX = whiteNoise.Results.tolX ;
tolFun = whiteNoise.Results.tolFun ;
f3DB = whiteNoise.Results.f3DB ;
dataPlot = whiteNoise.Results.dataPlot ;
% check number of input
narginchk(4,8)% narginchk(LOW,HIGH) throws an error if nargin is less than LOW or greater than HIGH


%% Get envelop parameters
dt = median(diff(t));

h1=fdesign.lowpass('N,F3dB',8,f3DB,1/dt);
d1 = design(h1,'butter');
Y = filtfilt(d1.sosMatrix,d1.ScaleValues, abs(hilbert(y)));%Y = filtfilt(B, A, X) filters the data in vector X with the filter described by vectors A and B to create the filtered data Y.
Y = Y./max(abs(Y));
options=optimset('Display','off','TolX',tolX,'TolFun',tolFun);
coeff1=lsqcurvefit(@(para,t)
Envelop(para,t),guessEnvelop,t,Y,[0.01,0.01,0.1],[3,3,100],options);

eps = coeff1(1);
T90 = coeff1(2);
tn = coeff1(3);

%% Get stationary perameters for the spectrum
E =Envelop(coeff1,t);
x = y./E; % there may be better solution than this one, but I don't have better idea right now.
x(1)=0;

% calculate the POWER SPECTRAL DENSITY
[PSD,freq]=pwelch(x,[],[],[],1/median(diff(t)));%%
```

```
coeff=lsqcurvefit(@(para,t)
KT(para,freq),guessKT,freq,PSD,[0.01,0.01,1],[5,5,100],options);
zeta = coeff2(1);
sigma = coeff2(2);
fn = coeff2(3);

%% dataPLot (optional)
if strcmpi(dataPlot,'yes'),
   spectra = KT(coeff2,freq);

   figure
%     subplot(211)
   plot(t,y./max(abs(y)),t,Envelop(coeff1,t),t,Y,'r')
   legend('Typical Earthquake','Broadband Tuned','White Noise')
   title([' T_{90} = ',num2str(coeff1(2),3),'; \epsilon = ',...
      num2str(coeff1(1),3),'; t_{n} = ',num2str(coeff1(3),3)]);
   xlabel('time (s)')
   ylabel('h(t,x)');
    axis tight ;
   figure;
%     subplot(212)
   plot(freq,PSD,freq,spectra,'r')
   legend('Measured','Fitted envelop')
   legend('Typical Earthquake','Broadband Tuned')
   title([' \zeta = ',num2str(coeff2(1),3),';  \sigma = ',...
      num2str(coeff2(2),3),';  f_{n} = ',num2str(coeff2(3),3)]);
   xlabel('frequency (Hz)')
   ylabel('H(w,x)')
    axis tight
xlim([0 10]);
   set(gcf,'color','w')
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% NESTED FUNCTIONS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

 function E = Envelop(para,t)
    eps0 = para(1);
    eta0 = para(2);
    tn0 = para(3);
    b = -eps0.*log(eta0)./(1+eps0.*(log(eta0)-1));
    c = b./eps0;
    a = (exp(1)./eps0).^b;
    E = a.*(t./tn0).^b.*exp(-c.*t./tn0);
 end
 function S = KT(para,freq);
    zeta0 = para(1);
    sigma0 = para(2);
    omega0 = 2*pi.*para(3);
    w =2*pi*freq;
    s0 = 2*zeta0*sigma0.^2./(pi.*omega0.*(4*zeta0.^2+1));
    A = omega0.^4+(2*zeta0*omega0*w).^2;
    B = (omega0.^2-w.^2).^2+(2*zeta0*omega0.*w).^2;
    S = s0.*A./B; % single sided PSD
 end
end
```

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