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CANDIDATE'S DECLARATION

I do hereby certify that the work presented in this report entitled "Study of Tail Equivalent Linearization method" in partial fulfillment of the requirement for the award of the degree of "Master of Technology" in structural Engineering, submitted in the Department of Civil Engineering, Delhi Technological University, is an authentic record of my own work under the supervision of Mr. G.P. Awadhiya, Associate Professor Department of Civil Engineering.

I have not submitted this matter for the award of any other degree or diploma.

Date: July 31,2014

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2K12/STE/06
M.Tech. (Strutures)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

This study explaining the tail-equivalent linearization method, to the case of a nonlinear structure subjected to stochastic excitations. with reference to Fujumura Der Kiureghian (2007), this method works on a discrete representation of the stochastic inputs and the first-order reliability method. the genetics of TELM is first order reliability method, and each component of the Gaussian excitation is expressed as a linear function of standard normal random variables. For a specified response threshold of the nonlinear system at a specified time, the tail equivalent linear system is defined in the standard normal space by matching the "design point" of the equivalent linear and nonlinear responses. This leads to the identification of the TELS in terms of a unit-impulse response function for each component of the input excitation. tail equivalent linearization method is a new, non-parametric linearization method for nonlinear random vibration analysis. This method is overcome the inadequacy of conventional equivalent linearization method.

our objectives are investigation and thorough understanding of analysis of stochastic non- linear system by tail equivalent linearization method as well computation of certain non-linear response characteristics. The excitations, that will be studied, are stationary Gaussian processes. These processes can be white noise processes. the primary motive of this study to present thorough investigation of nonlinear stochastic dynamic analysis using TELM (tail equivalent linearization method), and simultaneously we generate a random excitation by use of white noise simulation. we generate a computational program for white noise Gaussian process simulation. further more study presented on method of random vibration analysis especially on equivalent linearization method and also gives brief review on reliability analysis of structure, i.e. first order reliability method and second order reliability method. in this section describe the problems of interest characterized by simple geometric forms for linear systems subjected to Gaussian excitation. Approximate solutions for such problems are obtained by use of the first- and second-order reliability methods (FORM and SORM). Examples are solve for demonstrate the approach.

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CHAPTER 1- INTRODUCTION

1.1 General

In many practical applications, due to the high intensity nature and complex nature of environmental loads such as earthquakes, wind loads, and sea waves, the systems subjected to these loadings may experience excessive stress or displacements that results in elastic or even hysteretic behavior. This is particularly the case under high intensity random excitation. Under these conditions, it is difficult to obtain the closed form solution for dynamic response of a nonlinear system. In this case nonlinear random vibration methods are the best methods in the analysis of the structures under sever loads associated with natural hazards.

Random vibration for linear structures uses the superposition principle. However, this advantage is not applicable for nonlinear systems, but there are ways to transform a nonlinear system to an equivalent linear system that can benefit from this advantage.

Over the past few decades, a number of methods for nonlinear random vibration have been developed. These include methods using the Fokker–Planck equation, stochastic averaging, moment closure, perturbation, and equivalent linearization. Recent accounts of these methods can be found in the text by N.C. Nigam [1] Among these, equivalent linearization method (ELM) which is widely used because of its simplicity and applicability to general, In ELM the equivalent system is selected by minimizing the mean-square error between the responses of the non- linear and the linear systems based on the assumption of Gaussian response for the nonlinear system. Since the Gaussian assumption is not valid for high nonlinear systems, although the accuracy of the method is good in estimating the mean-square response, the probability distribution can be far from correct, particularly in the tail region. Thus estimates of response statistics such as crossing rates and first-passage probability, issues of which are of particular interest in reliability analysis can be grossly inaccurate at high thresholds. Another approach is Monte Carlo simulation, which is generally applicable, but is computationally demanding.

. Origin of the method lies in the first-order reliability method (FORM), which aims to solve this class of problems with good accuracy in the tail region. To overcome the shortcomings of the conventional ELM, Fujimura and Der Kiureghian (2007) presented tail equivalent linearization method (TELM), which uses the advantages of first order reliability method (FORM).

TELM is also an equivalent linearization method. However, instead of defining the linear system by minimizing the mean-square error in the response, it is defined by matching the tail probability of the linear response to a first-order approximation of the tail probability. For this reason, the name Tail-Equivalent Linearization Method (TELM) is used

In TELM, the input process is discretized and represented by a set of standard normal random variables. Each response threshold defines a limit state surface with the "design point" being the point on the surface that is nearest to the origin. Design point in FORM is the point on a limit-state surface that is nearest to the origin when the random variables are transformed to the standard normal space.

Linearization of the limit-state surface at this point uniquely defines a linear system, denoted as Tail-Equivalent Linear System, TELS. Previous study shows that design point on limit state surface of linear system and nonlinear system is same. The tail probability of the TELS response for the specified threshold is equal to the first-order approximation of the tail probability of the nonlinear system response for the same threshold.

Once the TELS is defined for a specific response threshold, methods of linear random vibration analysis are used to compute various response statistics, such as the mean crossing rate and tail probabilities of local and extreme peaks. The method has been developed for application in both time, Fujimura and Der Kiureghian (2007-09), and frequency domain, Garre and Der Kiureghian (2010), and it has been applied for inelastic structures as well as structures experiencing geometric nonlinearities.

1.2 Objective and scope of the study

The objectives are investigation and thorough understanding of analysis of stochastic non- linear system by Tail Equivalent linearization method as well computation of certain non-linear response characteristics.

This study to present thorough investigation of Nonlinear stochastic dynamic analysis using TELM (Tail Equivalent linearization method), and influence of various parameters on the tail equivalent linear system, such as discrete representation of stochastic excitation, characterization of linear system etc. Apart from TELM for the use of synthetic ground motion generating synthetic ground motions by use of white noise Gaussian process. For studying of TELM we want to basic idea about random vibration analysis and Methods of Structural Reliability Analysis.

TELM is based on first order reliability method and equivalent linearization method of random vibration.

TELM is combination of FORM and ELM means reliability analysis and random vibration analysis. In this study we give brief review of both the methods.

The method was initially developed in the field of earthquake engineering, where a discretization in time domain is convenient. A corresponding definition of the tail-equivalent linear system was then obtained in terms of its unit impulse-response function.

A number of applications of this method in the civil engineering field have been investigated for both stationary and non-stationary problems, single and multi-degree-of-freedom systems, and a variety of non-degrading, hysteretic material models, demonstrating its validity and accuracy.

1.3 Organization of Report

This report is organized into nine chapters. In first chapter of the section is a short review of TELM and importance of this method are given which we discussed in preceding sections. This gives a idea about the method of structural analysis of system. For understanding the TELM .We required some good knowledge of random vibration analysis and reliability analysis of structure so that we also require a review of both method of analysis first after that we go on TELM. TELM is based on FORM i.e. Reliability analysis and after obtaining the TELS we apply random vibration analysis on that and get required characteristics.

This report is written in this format, after a review of previous literature on TELM and other linear nonlinear analysis of structural system, are discussed in 2^{nd} chapter, we started from random vibration analysis to TELM. In this sequence we briefly study the random vibration analysis of structure is discussed in chapter 3. It has different methods of analysis in the form of short note and Gaussian processes are discussed. In this chapter also describe the white noise and their simulations by use of programming on macro in MS excel with generating the random variable of zero mean and unit variance.

After this we go on chapter 4 which has second part of our study that is reliability analysis of structural systems. In this we try to introduce basic of TELM, by means of first order reliability analysis and second order reliability method of reliability analysis. Here describe FORM method which is a parent method of TELM. In this method and TELM, more similarities are available, for example in FORM And TELM first step is dicretization of input excitation.

For the knowledge of FORM discussed in previous chapter are used in the chapter 5 In this section of report describe the TELM, which is our interest of study. Chapter 5 is also sub divided in deferent section for make a simple understanding of TELM, TELM is non-parametric method it's not depends on the uncertainty of system parameters. We apply FORM method on system and obtained the TELS and after obtaining equivalent system we apply random vibration analysis. That is the reason we introduce chapter 6 which contain only random vibration analysis on TELS. It's a last step of TELM.

For demonstrate the method we work on some example which is given in this chapter 7 mainly two examples are solved one of them deals with the response of a linear oscillator to two different non Gaussians excitations. Second example is determine the characteristic of a column of six storey building by the use of FORM and main objective of this examples are to demonstrate the applications of FORM an their accuracy.

CHAPTER 2 LITERATURE REVIEW

Kazuya Fujimura, Armen Der Kiureghia [1]**,** presented tail equivalent linearization method which uses the advantages of first order reliability method (FORM). In this method stochastic excitation is discretized and represented in terms of a finite set of standard normal random variables. TELM is new, non-parametric linearization method for nonlinear random vibration analysis. For a specified response threshold of the nonlinear system, the equivalent linear system is defined by matching the "design points" of the linear and nonlinear responses in the space of the standard normal random variables obtained from the discretization of the excitation. Due to this definition, the tail probability of the linear system is equal to the first-order approximation of the tail probability of the nonlinear system, for this property motivating the name Tail-Equivalent Linearization Method (TELM). He is shown that the equivalent linear system is uniquely determined in terms of its impulse response function in a non-parametric form from the knowledge of the design point. He is examine the influences of various parameters on the tail-equivalent linear system, presents an algorithm for finding the needed sequence of design points, and describes methods for determining various statistics of the nonlinear response, such as the probability distribution, the mean level-crossing rate and the first-passage probability. Applications to single- and multi-degree-of-freedom, non-degrading hysteretic systems illustrate various features of the method, and comparisons with results obtained by Monte Carlo simulations and by the conventional equivalent linearization method (ELM) demonstrate the superior accuracy of TELM over ELM, particularly for high response thresholds.

Luca Garrè , Armen Der Kiureghian [2] extended the previous work on the Tail-Equivalent Linearization Method (TELM) to the frequency domain. The extension defines the Tail-Equivalent Linear System in terms of its frequency-response function. This function is obtained by matching the design point of the nonlinear response with that of the linearized response. The proposed approach is particularly suitable when the input and response processes are stationary, as is usually the case in the analysis of marine structures. When linear waves are considered, the Tail-Equivalent Linear System

possesses a number of important properties, such as the capability to account for multi-support excitations and invariance with respect to scaling of the excitation. The latter property significantly enhances the computational efficiency of TELM for analysis with variable sea states. Additionally, the frequency-response function of the Tail-Equivalent Linear System offers insights into the geometry of random vibrations discretized in the frequency domain and into the physical nature of the response process. The proposed approach is applied to the analysis of point-in-time and first-passage statistics of the random sway displacement of a simplified jack-up rig model. A basic requirement of TELM is the discretization of the input excitation in terms of a finite set of standard normal random variables. In fact, the equivalence in TELM is established in the space of these random variables by matching the design points of the linear and nonlinear responses, which are points on their respective limit-state surfaces with minimal distance to the origin in the standard normal space. The method was initially developed in the field of earthquake engineering, where a discretization in time domain is convenient. A corresponding definition of the tail-equivalent linear system was then obtained in terms of its unit impulse-response function. A number of applications of this method in the civil field have been investigated for both stationary and non-stationary prob- lems, single and multi-degree-of-freedom systems, and a variety of non-degrading, hysteretic material models demonstrating its validity and accuracy.

In marine problems, it is customary to define the wave-induced excitation in the frequency domain. In that case, identification of the TELS in terms of a frequency-response function is advantageous, as it facilitates frequency-domain random vibration analysis, In the present paper, TELM is developed for a frequency-domain discretization of the input excitation and it is shown that the frequency- response function of the TELS can be directly computed from the design point of the nonlinear response. An extension of the Tail-Equivalent Linearization Method (TELM) for nonlinear stochastic dynamic analysis in the frequency domain is presented. It is shown that, when the input excitation is discretized in the frequency domain, the frequency-response function (FRF) defining the tail-equivalent linear system is directly obtained from the design point of the nonlinear response. Formulations are derived for the computation of the modulus and phase of the FRF. The so-found equivalent FRF possesses a number of properties, such as the capability to account for multi-support excitations and invariance with respect to scaling of the excitation when linear waves are considered. The latter property is particularly useful with regard to long-term analyses of the response of marine structures, for which several sea states must be considered. The equivalent FRF

encompasses all the nonlinearities included in the equation of motion. In the present simplified application the nonlinear drag term of the Morison's equation, relative velocity with respect to rig legs and inundation effects were included. Additionally, the FRF offers insight into the geometry of the random vibrations discretized in the frequency-domain.

Armen Der Kiureghian and Kazuya Fujimura, [3] A new alternative approach for computing seismic fragility curves for nonlinear structures for use in PBEE analysis is proposed. The approach makes use of a recently developed method for nonlinear stochastic dynamic analysis by tail-equivalent linearization. The approach avoids repeated time-history analysis with a suite of scaled, recorded ground motions. Instead, the ground motion is modeled as a stochastic process and, after determining the TELS for each response threshold, simple linear random vibration analyses are performed to compute the fragility curve. In the present application, the same stochastic model was used for all intensity levels. However, this is not necessary. One can easily vary the parameters of the stochastic model with the intensity level to more realistically characterize high-intensity motions. In doing this, since the TELS remains invariant of the scaling and frequency content of the excitation, one will only need to change the excitation model in the linear random vibration analysis of the TELS for different intensity levels.

While offering a viable alternative for fragility analysis, the proposed method has its limitations. For example, at the present time it is only applicable to non-degrading systems, and only one component of ground motion was considered in the present application. Furthermore, response gradient computations are required and, therefore, a dynamic analysis code with this capability must be used. Nevertheless, the proposed method offers an alternative to a type of analysis for which few other viable alternatives are presently available.

Sanaz Rezaeian and Armen Der Kiureghian, [4] described in there report stochastic modeling and simulation of ground motion time-histories for use in response-history or stochastic dynamic analysis. Ultimately, this research benefits the emerging

field of performance-based earthquake engineering (PBEE) by providing a convenient method of generating synthetic ground motions for specified design scenarios that have characteristics similar to those of real earthquake ground motions. The major developments and findings of this study are summarized as follows:

A new site-based, fully nonstationary stochastic model to describe earthquake ground motions is developed. The model is based on time modulation of the response of a linear filter with time-varying characteristics to a discretized white-noise excitation. It is concluded that for a typical strong ground motion the filter frequency can be represented by a linear function, whereas the filter-damping ratio can be represented by a constant or a piece-wise constant function.

Acceleration time-histories obtained by simulating the stochastic model are high-pass filtered (to achieve zero velocity and displacement residuals. The selected filter is a critically damped oscillator. The oscillator frequency determines the level of high-pass filtering and helps to avoid overestimation of simulated response spectrum ordinates at long periods.

The stochastic ground motion model has advantages over existing models:

(a) The stochastic model represents both the temporal and spectral non-stationary (characteristics of real earthquake ground motions. As a result, the modulating function characterizes the variation of the intensity in time, whereas the time-varying filter describes the evolving frequency content.

(b) The model has a small number of parameters with physical interpretations. These parameters can be as few as six, with three parameters controlling the evolving intensity of the motion, two parameters controlling the evolving predominant frequency of the motion, and one parameter controlling the bandwidth.

(c) Modeling is done entirely in the time-domain.

(d) The discretized form of the model facilitates digital simulation as well as nonlinear (stochastic dynamic analysis.

(e) Simulation of a synthetic ground motion for specified model parameters is simple and (requires little more than generation of standard normal random variables, their multiplication with deterministic time-varying functions, and post-processing through a high-pass filter. It maintains the natural variability of real ground motions.

Caughey TK [5] proposed generalized to the case of nonlinear dynamic systems with random excitation. The method is applied to a variety of problems, and the results are compared with exact solutions of the Fokker-Planck equation for those cases where the Fokker-Planck technique may be applied. Alternate approaches to the problem are discussed including the characteristic function.

A. Der Kiureghian, [6] The geometry of random vibration problems in the space of standard normal random variables obtained from discretization of the input process is described. For linear systems subjected to Gaussian excitation, simple geometric forms, such as vectors, planes and ellipsoids, characterize the problems of interest. For non-Gaussian responses, non-linear geometric forms characterize the problems. Approximate solutions for such problems are obtained by use of FORM and SORM. This article offers a new outlook to random vibration problems and an approximate method for their solution. Examples involving response to non-Gaussian excitation and out-crossing of a vector process from a non-linear domain are used to demonstrate the approach.

Given a discrete representation of the input process in terms of standard normal random variables, it is shown that many statistical quantities of interest in random vibrations can be represented in geometric form in the standard normal space. These interpretations offer a new outlook to random vibration problems and potentially provide new tools for the approximate solution of non-Gaussian or non-linear problems. In this article, solution methods by FORM and SORM were explored. Possibilities for developing efficient simulation methods that exploit the geometric forms also exist. The numerical examples presented in this article indicate that FORM and SORM can be effective methods of solution, but they should be used with caution.

Heonsang Koo, Armen Der Kiureghian, Kazuya Fujimura [7]**,** provided A key step in finding the design-point excitation, which is that realization of the input process that is most likely to give rise to the event of interest. It is shown in this paper that for a non-linear elastic SDOF oscillator subjected to a Gaussian white-noise input, the design-point excitation is identical to the mirror image of the free-vibration response of the oscillator when it is released from the target threshold. With a slight modification, this result is extended to problems with non-white and non-stationary excitations, as well as to hysteretic oscillators. For these cases only an approximation to the design point is

obtained. If necessary, the approximation can be used as a ‘warm’ starting point in an iterative algorithm to obtain the exact design point. The paper also introduces a simple and accurate method for estimating the mean up- crossing rate of random vibration response by FORM analysis.

Yan-Gang Zhao, Tetsuro Ono, [8] stated that the FORM/SORM accuracy is generally dependent on three parameters,

a. The curvature radius at the design point,

b. The number of random variables and

c. The first-order reliability index.

In this literature, the ranges of the three parameters for which FORM/SORM is accurate enough are investigated.

For practical application of FORM/SORM, a general procedure is proposed which includes three steps:

1. Point fitting limit state surface,
2. computation of the total principal curvatures Ks ,
3. failure probability computation according to the range of Ks.

The procedure proposed in this paper can be used only for limit state surfaces that have only one design point, a restriction that also applies to other FORM/SORM methods. Otherwise local convergence may occur, and error results may be yielded, and the ranges of applicability of FORM/SORM cannot be used in the case that the curvatures at the design point have different signs and extremely unevenly distributed.

M. Ababneh*, M. Salah, K. Alwidyan, in his paper, a comparison between the optimal linear model and Jacobian linearization technique is conducted. The performance of these two linearization methods are illustrated using two benchmark nonlinear systems, these are inverted pendulum system; and Duffing chaos system. Linearization of nonlinear dynamical systems is a main approach in the designing and analyzing of such systems. Optimal linear model is an online linearization technique for finding a local model that is linear in both the state and the control terms.

Fayçal Ikhouane, Víctor Mañosa, José Rodellar, They Bouc-Wen model, widely used in structural and mechanical engineering, gives an analytical description of a smooth hysteretic behavior. It may happen that a Bouc-Wen model presents a good matching with the experimental real data for a specific input, but does not necessarily keep significant physical properties that are inherent to the real data, independently of the exciting input. This literature presents a characterization of the different classes of Bouc-Wen models in terms of their bounded input-bounded output stability property, and their capability for reproducing physical properties inherent to the true system they are to model.

Chapter 3 Brief review of random vibration analysis

3.1 Introduction

3.1.1 General

When an excitation function applied to a structure has an irregular shape that is described indirectly by statistical means, we called of a random vibration.

Such a function is usually described discrete or continuous function of the existing frequencies, in a manner similar to the description of the function by Fourier series. A random variable is a variable that takes on numerical values according to a chance process. Random variable generally two types

Discrete random variable, is a countable values i.e. number of coursed selected by the students in university.

Continuous random variable, all values in intervals like (0,1) i.e. height of randomly selected adult candidates in range of 1 to 2 meter.

In structural dynamics, the random excitations most often encountered are either motion transmitted through the foundation or acoustic pressure both of these types of loadings are generated by explosions occurring vicinity of the structure. Common sources of these explosions are construction work and mining.

Other type of loading, such as earthquake excitation, may also be considered random function of time. In this case the structural response is obtained in probabilistic terms using random vibration theory. A record of random vibration is a time function such a shown in figure.1. The main characteristics of such random function are that its instantaneous value cannot be predicted in a deterministic sense. The description and analysis or random processes are established in a probabilistic sense for which it is necessary to use tools provided by the theory of statistics. [9]

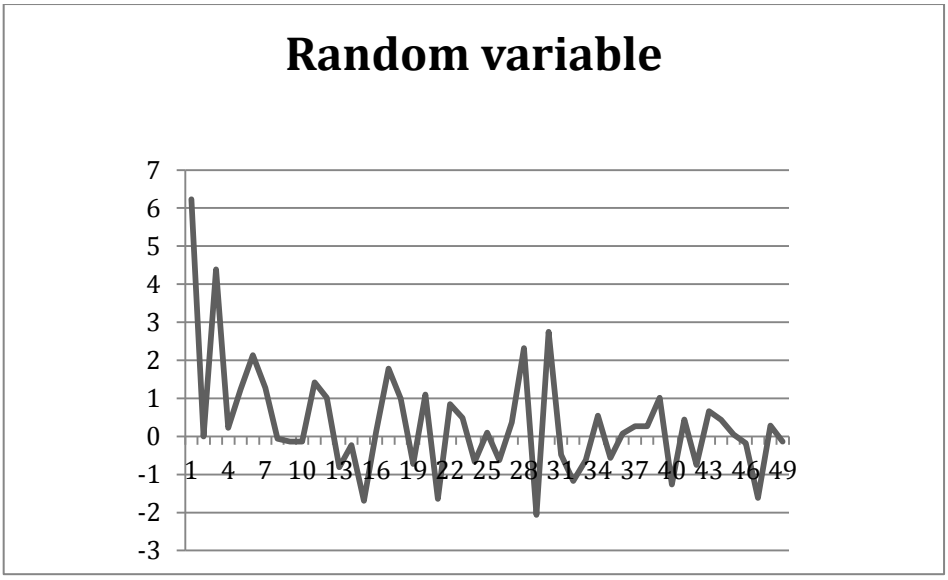

Figure 1 random function simulation

3.1.2 Statistical Description of Random Function

In any statistical method a large number of responses is needed to describe a random function. For example, to establish the statistics of the foundation excitation due to explosions in the vicinity of a structure, many records of type shown in fig 2 may be needed.

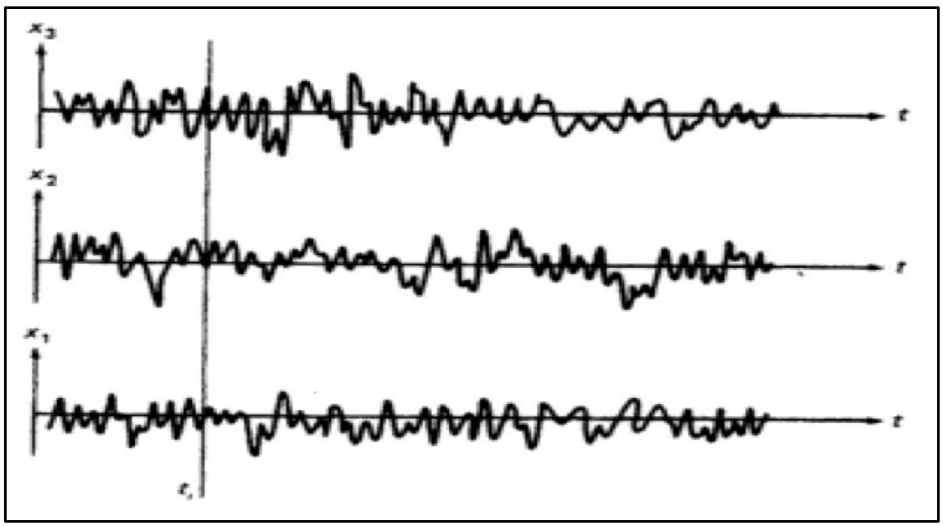

Figure 2 An ensemble of random function

Each record is called sample, and the total collection of samples an ensemble. To describe an ensemble statistically, we can compute at any time t_i the average value of the instantaneous displacements xi. If such averages do not differ as we select different values of t_i, then the random process is said to be stationary. In addition if the average obtained with respect to time for any member of the ensembles is equal to the average across the ensemble at an arbitrary time t_i, the random process is called ergodic. Thus In a stationary ergodic process a single record may be used to obtain the statistical description of a random function. We shall assume that all random process considered are stationary and ergodic. The random function of time shown ing fig has been recorded during and interval of time T. several averages are useful in describing such a random function. The most common are the mean value $\bar{x}$ which is defined as

$$\bar{x} = \frac{1}{T}\int_0^T x(t)dt \tag{3.1}$$

And the mean square value $\overline{x^2}$ defined as

$$\overline{x^2} = \frac{1}{T}\int_0^T x^2(t)dt \tag{3.2}$$

Both the mean and the mean square values provide measurements for the average value of the random function x(t). The measure of how widely the function x(t) differs from the average is given by its variance σ_x^2 defined as

$$\sigma_x^2 = \frac{1}{T}\int_0^T [x(t) - \bar{x}]^2 dt \tag{3.3}$$

When the expression under the integral is expanded and then integrated, we find that

$$\sigma_x^2 = \overline{x^2} - \bar{x}^2 \tag{3.4}$$

This means that the variance can be calculated as the mean square minus the square of the mean. Quite often the mean value is zero, in which case variance is equal to the mean square value. The root mean- square RMS_x of the random function x(t) is defined as

$$RMS_x = \sqrt{\overline{x^2}}$$

The standard deviation

$$\sigma_x = \sqrt{\overline{x^2} - \bar{x}^2}$$

3.1.3 Probability density function and Cumulative distribution function

Probability density function

Fig 2 shows as portion of a record of a random function x(t). Through the value x_1 & x_2, and then measure the corresponding time intervals Δt_i. The ratio given by

$$P(x_1 \leq x \leq x_2) = \frac{\Delta t_1 + \Delta t_2 + \cdots .. \Delta t_n}{T} \quad (3.5)$$

and calculated for the entire record length T, is the probability of x having the value between x_1 & x_2 at any selected time t_i during the random process.

Similarly the probability of x(t) being a smaller than a value of x can be expressed as

$$P(x)=P[x(t)<x]=\lim_{T\to\infty} \frac{1}{T}\sum_i \Delta t_i \quad (3.6)$$

Where the time interval Δt_i are now those for which the function x(t) as a value smaller than the specified x.

The function P(x) in equation (3.6) is known as the cumulative distribution function of the random function x(t). This function is plotted in fig-4 as a function of x. The cumulative distribution function is a monotonically increasing function for which

$$P(-\infty) = 0, 0 \leq P(x) \leq 1, P(\infty) = 1 \quad (3.7)$$

Now the probability that the value of the random variable is smaller than the value $x+\Delta x$ is denoted by $P(x+\Delta x)$ and that x(t) takes value between x and $x+\Delta x$ is $P(x+\Delta x) - P(x)$. This allows us to define the probability density function as

$$p(x)=\lim_{\Delta x\to} \frac{P(x-\Delta x)-P(x)}{\Delta x} = \frac{dP(x)}{dx} \quad (3.8)$$

Thus the probability density function p(x) is represented geometrically by the slope of the cumulative probability function P(x). The functions p(x) and P(x) are shown in that

fig (3.3) &(3.4) . From equation (3.8) we conclude that the probability that a random variable x(t) has a value between x and x+$\Delta x$is given by p(x) dx,where p(x) is the probability density junction.

$$P((x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} p(x)dx \quad (3.9$$

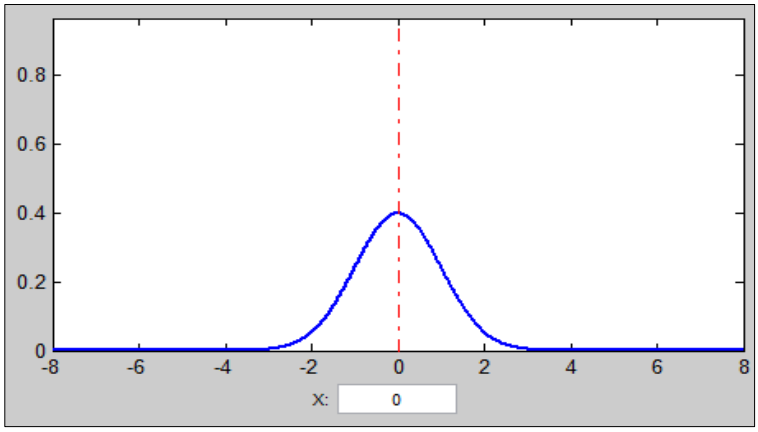

Figure 3 Probability density function

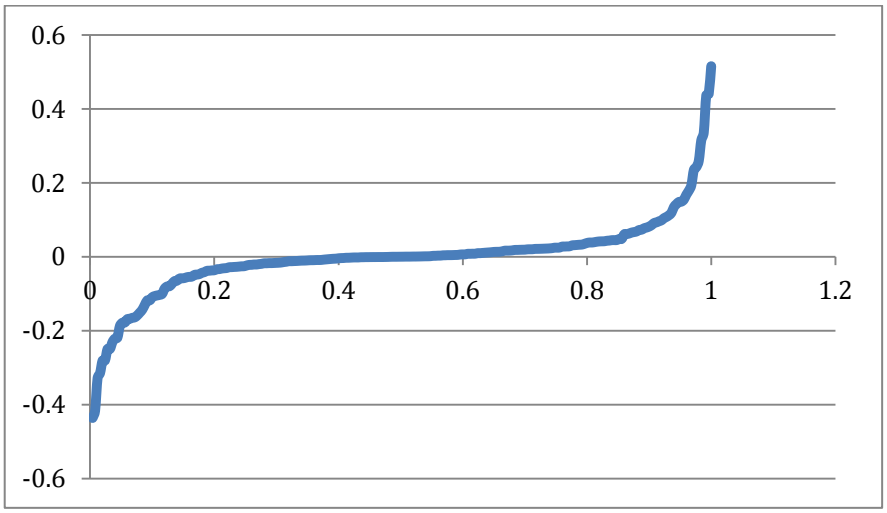

Figure 4 Cumulative probability function

3.1.4 Some Useful Probability Distributions

In this section, some probability distributions of continuous random variable and their properties, which are used in practical applications mostly, are presented briefly.

3.1.4.1 Normal (Gaussian) Distribution

The Normal or Gaussian probability density function of a random variable X is the one mostly used in practice. It is defined in general as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\frac{1}{2}(x-\bar{x})^2}{\sigma^2}} \tag{3.10}$$

In which $\bar{x}$ and σ^2 are respectively the mean and standard variation of X. The Corresponding CDF is calculated from:

$$P(x) = \int_{-\infty}^{\infty} p(\epsilon)\, d\epsilon = \phi\left(\frac{x-\bar{x}}{\sigma}\right) \tag{3.11}$$

where ϕ(-) is called as the Standard Normal Distribution function and its PDF is denoted by φ(-), which are defined:

$$\text{Standard Normal PDF} \qquad \varphi(x) = \frac{1}{\sqrt{2\pi}} \int e^{-x^2/2} \tag{3.12}$$

$$\text{Standard Normal CDF} \qquad \phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-u^2/2}\, du \tag{3.13}$$

If multivariate normal variables are involved in a process, then a multivariate normal PDF will be required. In this case, a vector process is used and the multivariate normal PDF is stated as,

$$\text{Multivariate Normal } PDF \qquad p(x) = \left(\frac{1}{2\pi}\right)^{p/2} \frac{1}{\sqrt{|\rho|}} e^{-\frac{x^2}{2}} \tag{3.14}$$

Where $\bar{X}$ is a vector of p-dimensional random variable, $\bar{x}$ is a vector of their realizations and χ^2 is a scalar calculated from the product

$$\text{Scalar:} \rightarrow \chi^2 = (\bar{x} - \bar{m})^T \rho^{-1} (\bar{x} - \bar{m}) \quad (3.15)$$

In which $\bar{m}$ is a vector of mean values and ρ is the covariance matrix of $\bar{x}$ and $|\rho|$ in (3.15) denotes the determinant of ρ.

These definitions are written:

Vector of Multivariate random variable: $\rightarrow \breve{\mathbf{X}} = \{\mathbf{X_1}, \mathbf{X_2} \ldots \ldots \mathbf{X_n}\}^{\mathbf{T}}$

Vector of realizations of $\breve{\mathbf{X}}$: $\rightarrow \breve{\mathbf{X}} = \{x_1, x_2 \ldots \ldots x_p\}^T$

Vector of mean values of $\breve{\mathbf{X}}$: $\rightarrow \breve{m} = \{m_1, m_2 \ldots \ldots m_p\}^T$

The covariance matrix ρ is defined as

Covariance matrix of $\breve{\mathbf{X}}$: $\rightarrow \rho = \begin{bmatrix} \sigma_1^2 & \sigma_{12\ldots} & \sigma_{1p} \\ \sigma_{21} & \sigma_2^2 & \sigma_{2p} \\ \sigma_{p1} & \sigma_{p2} & \sigma_p^2 \end{bmatrix}$ (3.16)

As it is seen from equation the diagonal terms of this matrix are the variances of the random variable X_i, for uncorrelated random variable, the off diagonal terms will be zero and the matrix becomes diagonal.

3.1.4.2 Lognormal Distribution

One other commonly used distribution in practice is the Lognormal Distribution. If the random variable X has a Normal distribution with the mean and variance, m_X and σ_x^2, then the random variable. $Y = e^X$ is said to be log normally distributed. It is written as

Exponential Function of X: $\rightarrow$ $Y = e^X$ and $X = \ln Y$

Using eq. the PDF of the random variable $\gamma = e^x$, Can be obtained as written

Lognormal PDF: $p(x)\frac{1}{\sigma_{x\sqrt{2\pi}}}\frac{1}{y}e^{-\frac{1}{2}\left(\frac{\ln y-mx}{\sigma_x}\right)^2}$ for (y>0) (3.17)

In the region of (y<0), PDF of the random variable will be zero, i.e. $f_y(y)=0$ for (y<0). The mean and variance of a lognormal random variable, are calculated from

Mean of the random variable Y=e^x: $\rightarrow m_y = e^{m_x}e^{\frac{\sigma_x^2}{2}}$

Variance of the random variable Y=e^x: $\rightarrow \sigma_y^2 = m_y^2\left(e^{\frac{\sigma_x^2}{2}}-1\right)$

If m_y and $\sigma_y$are given, then the variance and mean of X are calculated from the following statements

$$\sigma_x^2 = ln\left[1+(\frac{\sigma_Y}{\mu_Y})^2\right] \quad (3.18)$$

And

$$\mu_x = \left(ln\mu_y - \sigma_x^2/2\right) \quad (3.19)$$

3.1.4.3 Gamma Distribution

The Gamma Distribution represents the sum of r independent exponentially distributed random variable, and random variable that take always positive values. Its PDF and CDF functions are defined as written

Gamma Dist., PDF:$\rightarrow f_x(x) = \frac{\lambda}{\Gamma(r)}(\lambda x)^{r-1}e^{-\lambda x}$ if ($x\geq 0, \lambda > 0$) (3.20)

Gamma Dist., CDF: $F_x(x) = 1-\sum_{k=0}^{r-1}\frac{1}{k!}(\lambda x)^k(\lambda x)^k$ if ($x\geq 0, \lambda > 0$) (3.21)

In which $\Gamma(.)$ represents a Gamma function which is defined

Gamma function: $\rightarrow \Gamma(x) = \int_0^\infty e^{-u} u^{(x-1)}\, du$ (3.22)

The mean and variance of the Gamma distribution are calculated to be

$$\text{Mean: } \rightarrow \; m_x = \frac{r}{\lambda} \quad \text{and variance: } \rightarrow \; \sigma_x^2 = \frac{r}{\lambda^2}$$

The parameters r and λ are respectively the shape and scale parameters of the distribution. For different values of r and λ, different type of distributions are obtained. When (r=1), it gives the exponential distribution. If (r<1), Then the distribution is exponentially shaped and asymptotic to both horizontal and vertical axes. If (r>1), its shape is unimodal and skewed with the mode equals ($x_m = (r-1)/\gamma$). The skewness reduces with increasing value of r as it is seen from the coefficient of skewness, ($\gamma_1 = \frac{2}{\sqrt{r}}$). If (r=s/2) and ($\gamma = 1/2$), Then the gamma distribution becomes the χ^2 distribution with s degree of freedom. In engineering applications, Gamma distributions occur frequently in models of failure analysis, for rainfall studies since the variables are always positive and the results are unbalanced.

3.1.4.4 Rayleigh Distribution

The Rayleigh Distribution is used as a probability model describing the distribution of wind speed over 1-year period. It is often used for the probability model of the absolute value of two components of a random field, e.g., if X and Y are two independent normally distributed random variables, both with zero mean and variance equal to σ^2 and if we define a function $Z = \sqrt{X^2 + Y^2}$ then this function has a Rayleigh distribution with the parameter σ. It also describes the probability distribution of maxima of a narrow band random process with Normal distribution. The PDF and CDF of the Rayleigh distribution are given as

Rayleigh PDF: $\rightarrow f_x(x) = f(x) = \begin{cases} \frac{x}{\sigma^2}\exp(-\frac{x^2}{2\sigma^2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ (3.23)

Rayleigh CDF: $\rightarrow \ f_x(x) = \ 1 - \exp(-\frac{x^2}{2\sigma^2})$ (3.24)

In which σ is the only parameter of the distribution, which is equal to the standard deviations of the independent random variables X and Y with Normal distributions and zero mean. The mean and variance of the Rayleigh distribution are calculated to be

Mean: $\rightarrow m_x = \sigma\sqrt{\frac{\pi}{2}}$ (3.25)

Variance: $\sigma_x^2 = 2\sigma^2\left(1 - \frac{\pi}{4}\right)$ (3.26)

3.1.4.5 Gumbel Distribution

The Gumbel Distribution is usually used to model the distribution of the maxi- mum, or the minimum, of a number of samples or various distributions. It can also be used to find the probability that an extreme event, such as earthquake, flood or other natural disaster, will occur. The Gumbel distribution is also known as the Extreme Value Type I Distribution. It has two forms as one is for extreme maximum (Extreme Value Largest I) and one is for extreme minimum (Extreme Value Smallest I), which are respectively defined below.

Gumbel (EV Largest-I): $\rightarrow f_x(x) = \alpha e^{-\alpha(x-\beta)}e^{-\exp(-\alpha(x-\beta))}$

$F_x(x) = e^{-\exp(-\alpha(x-\beta))}$ for (-$\infty < x < \infty$) (3.27)

Gumbel (EV Smallest-I): $f_x(x) = \alpha e^{\alpha(x-\beta)}e^{-\exp(\alpha(x-\beta))}$

$F_x(x) = 1 - e^{-\exp(\alpha(x-\beta))}$for (-$\infty < x < \infty$) (3.28)

In which β is the location parameter and α is the scale parameter, which is defined ($\alpha > 0$). The Gumbel distribution supports the range of outcomes of the random

variable. X between ($-\infty < x < \infty$). The means and variances of both largest-I and smallest-I distributions are calculated from

$$\text{Mean}: \rightarrow \quad m_x = \beta + \frac{0.57722156649}{\alpha} \text{ (Largest-I)} \tag{3.29}$$

$$m_x = \beta - \frac{0.57722156649}{\alpha} \text{ (smallest –I)} \tag{3.30}$$

$$\text{Variance: } \rightarrow \sigma_x^2 = \frac{\pi^2}{6\alpha^2} \text{ (Largest-I and smallest – I)} \tag{3.31}$$

The value (0.57722156649) in (3.30) is the Euler's constant.

3.1.5 Correlation

Correlation is a measure of the dependence between two random processes. Consider the two records shown in fig.5

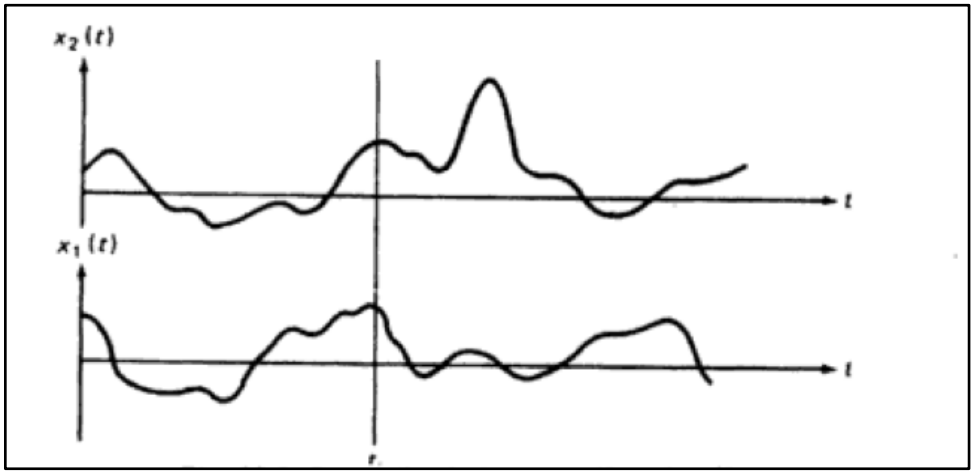

Figure 5 Correlation between $x_1(t)$ and x_2

Multiplying the coordinates of these two records at each time ti and computing the average over all values of t calculate the correlation between them. It is evident that the

correlation so found will be larger when two records are similar. For dissimilar records with mean zero, some products will be positive and other negative. Hence their average product will approach zero.

3.2 Random Processes

The theory of random processes has evolved as a generalization of the concept of random variable. In many problems the outcome of a event is not a real number but a function of one or more parameters, such as time or space or both, In such cases the outcome of each trial is called a realization or a sample function and the collection of all possible functions is called the ensemble of the random process. The random processes have found increasing application as models of a large class of natural phenomena. Some examples of phenomena modeled as random processes are [10]

1. Particles in suspension undergoing Brownian motion as function of time,
2. Ground motion a point during earthquakes as function of time,
3. Unevenness of road surface as a function of distance along the center line, and
4. The pressure field due to jet noise as function of both the time and space coordinates

As a brief definition, a random process is an infinite collection of grasps of an random variable. In a similar way to the definition of a random variable, a random process is a mapping from the sample space into an ensemble of time functions known as sample functions. The random variable X(x, t) for a fixed random x value, say x_1, is a specific time signal that it is called as the realization of the random variable. X(x, t) at $x = x_1$, which is denoted by $x_1(t)$. For a fixed time, say t_1, the random variable. $X(x, t_1)$ is a time-independent random variable, that probability principles are applied. For both fixed values of x and t, say ($x = x_1$ and $t = t_1$), the random variable. X(x, t) will be a real number with the value of $X(x_1, t)$. The ensemble of all realizations of the time-dependent random variable, represents the stochastic process that we use the notation X(t) to indicate it, disregarding its dependence on the outcome x. Such an ensemble, which represents a stochastic process, is shown in Fig.6 with four realizations, or samples, $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$. As indicated above, a stochastic process represents a single number, a time function, a random variable and a process with time function and random variable Thus,

1. if x and t are both fixed ($x = x_1$ and $t = t_1$), then $X(x_1, t_1)$ is a single number,
2. if x is fixed ($x = x_1$) and t is a variable, then $X(x_1, t)$ is a time function as $x_1(t)$,
3. if x is a variable and t is fixed ($t = t_1$), then $X(x, t_1)$ is a random variable at $t = t_1$,
4. if x and t are both variables then X(x, t) is a stochastic process.

If we consider an infinite number of samples, at a specific time, say $t = t_1$ as shown in Fig.6, the stochastic process will be a continuous random variable. With the outcomes x, as $(x_1, x_2, \ldots, x_n)$ where (n!1).

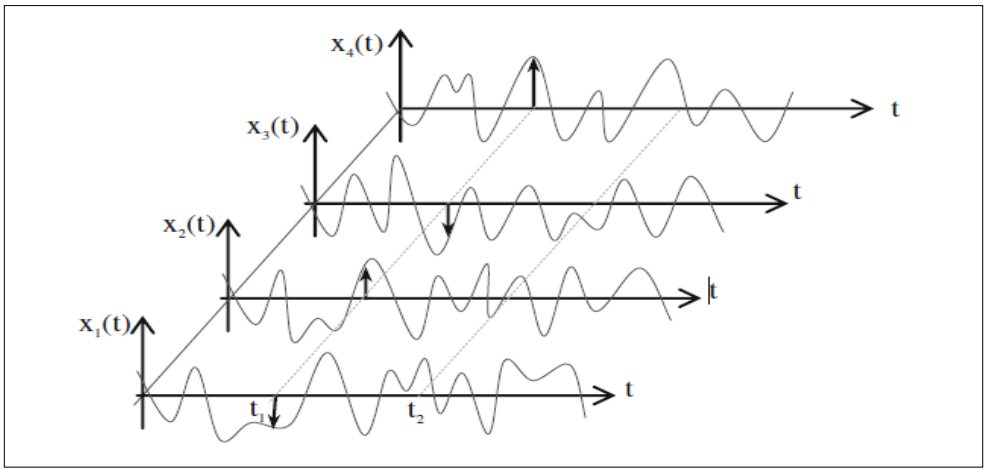

Figure 6 Ensemble of time-dependent random variable

3.2.2 White Noise

3.2.2.1 Introduction

In random Vibrations, white noise is extensively used to model excitations with a broad-banded frequency spectrum. The name '*white noise*' comes from the fact that its average power is uniformly distributed in frequency, which is a characteristic of white light.

This it has a constant power spectral density. This means that all frequencies contribute equal amount of energy to stochastic process up to infinite frequencies. Consequently, a white noise process has infinite variance. The fact that the infinitely high frequencies contribute to the process implies that it has zero memory [10].

When the excitation is a white noise process, the response generally is a *Markov process*. So, if a system is nonlinear or if multiplicative random excitations are present, or both, then, mathematically exact solution is not always obtainable. When such an exact solution is obtained, it is usually based on the assumption that system response is a Markov stochastic process or related to Markov process in some sense.

3.2.2.2 Gaussian White Noise Simulation on VBA(Macro)

A class of probabilistic model can be used to model random process. There are three basic approaches:

The model takes form of a non process like the Gaussian process Markov and the parameters of the model are estimated by using available data from actual records.

Somewhat similar to the first, sample function of the random process are generated at discrete values of the indexing parameters by passing a sequence of a random number through appropriately design filter. The filter weights, which are the parameters of the model, are estimated by making use of the actual observation of the random phenomenon.

This is non parametric in nature in the interest is mainly limited to the estimation of few order statistics of the random process.

Here we take first one for generating the white noise simulation and proceeding section we also modeling by third approach.

3.2.2.2.1 Random Number Generation

The most important step in application of Monte Carlo simulation is the generation of random number corresponding to prescribe probability distribution functions. Automatic generation of random number is most effectively and efficiently done on the digital computer.

The 1^{st} step in the generation of random number associated with the specific probability distribution function is to generate random numbers corresponding to the uniformly distribution random variable over the interval 0 to 1.

Let X be the random variable with the given probability distribution functions $F_x(x)$.

We are required to generation random numbers which can serve as a sample values of random variable X. it can be easily verified that the transform random variable given by the functional relation U= $F_x(x)$ is uniformly distributed over the interval 0 to 1 .
If $[u_n]= [u_1,u_2,u_3……u_n]$ is a set of sample value of U obtained by some procedure, the corresponding set of value $[x_n] =[x_1.x_2,x_3……x_n]$ obtained by the inverse transformation $xi=F_x^{-1}(ui)$ will be random numbers associated with the random variables X. Thus the

generation of uniformly distributed random number between 0 to 1 is most basic to generation of random numbers with general probability distribution.
The recursive relation which is often used in for this purpose is

$$X_{i+1}=(aX_i+b)(\text{modulo } m)= (aX_i+b)/m \qquad (3.32)$$

a , b & m are non negative integers
if K_i= integer part of the ratio $(aX_i+b)/m$.
then the corresponding residue of the modulus m is

$$X_{i+1}= aX_i+b-mK_i \qquad (3.33)$$

Divided the value obtained in eq. (3.33) by the modulus m, we obtained

$$U_{i+1} = X_i+1/m$$

Which form a set of uniformly distributed random number in the interval (0,1)
In the computerized procedure mentioned above the random are generated in a purely deterministic way. Hence are not strictly random. They can be duplicated exactly. For this reason they are called pseudo random numbers. The generated pseudo random number sequence is periodic with the period being less than the modulus m. in the Monte Carlo simulation method the period of random sequence should be greater than the number of random numbers that will be used in the simulation procedure. This will insure the randomness of the sequence in the particular application. The large value of m is needed for sufficiently large period. It can be show that the number generated with the large m appear to be uniformly distributed and statistically independent (Knuth.1969) satisfactory results have been observed (Rubinstein 1981) with $m=2^{35}$, a= 2^7+1 & b=1 for binary computers.

Another common recursive relation for generation uniform random numbers is the congruently generator defined by

$$X_{i+1} = ax_i * (\text{modulo } m) \quad (3.34)$$

$$\& \quad u_i = x_i/m \quad (3.35)$$

Equation (3.34) & (3.35) are used in the IBM system 360 as a uniform random number generator with

$$a=16808 \quad \& \quad m = 2^{31}-1.$$

3.2.2.2.2 White Noise Simulation

To generate sample function which approach a stationary Gaussian white noise process, first a sequence of pairs of statically independent random numbers
$[u_{2n}] = [u_1, u_2, u_3 \ldots\ldots u_{2n-1}, u_{2n}]$ all of which have a uniform probability distribution over the range $0 \leq u \leq 1$ is generated.
Given this sequence $[u_{2n}]$ of uniform random variants one can construct a new sequence of pairs of statically independent random numbers

$$[x_{2n}] = [x_1, x_2, x_3 \ldots\ldots x_{2n-1}, x_{2n}]$$

using the relation

$$X_{2i-1} = (-2\ln u_{2i-1})^{1/2} \cos(2\pi u_{2i}) \qquad i=1,2,3\ldots\ldots n \quad (3.36)$$

$$X_{2i} = (-2\ln u_{2i-1})^{1/2} \sin(2\pi u_{2i}) \qquad i=1,2,3\ldots\ldots n \quad (3.37)$$

It can be shown that the sequence $[x_{2n}]$ from independent samples of the Gaussian distribution with mean 0 & variance 1(Box & Muller-1958, Franklin-1965].
A sample function $a_1(t)$ can now be constructed by assigning the values $x_1, x_2, x_3 \ldots\ldots x_n$ to n successive ordinate spaced at equal intervals Δt along a time abscissa and by joining them by straight lines implying linear variation of the ordinates over each time of

interval. The time scale is chosen such that the initial time is a uniform random variable in the time interval $0<t<\Delta t$

Taking Fourier transforms of Rww(t) give the power spectral density function of the random process .

The time interval may be chosen sufficiently small to approximately generate the white noise process to within a given tolerance to any desired frequency.

3.2.2.2.3 Generate a program on VBA(macro) using excel for Gaussian white noise

In this section describe the coding for program on VBA for simulation of Gaussian white noise of zero mean and unit variance.

For example we take 10,000 random numbers in the programing we change the number of random variable easily how much we required.

For doing programing on excel sheet we go to developer and record the macro file and in the sheet one workbook wizard we type our program which is given below:

```
Sub randoma()
a = 2 ^ 7 + 1
b = 1
m = 2 ^ 35
xi = 1
For x = 1 To 100000
xiplus = (a * xi + b) / m
ki = Int(xiplus)
xiplus1 = a * xi + b - m * ki
uiplus1 = xiplus1 / m
xi = xiplus1
Sheet1.Cells(x, 1) = uiplus1

Next x
For x = 1 To 100000 Step 2
```

```
t = Sheet1.Cells(x, 1)
s = Sheet1.Cells(x + 1, 1)



x2iminus = ((-2 * Log(t)) ^ 0.5) * Cos(2 * (3.141592654) * s)
x2i = ((-2 * Log(t)) ^ 0.5) * Sin(2 * (3.141592654) * s)

Sheet1.Cells(x, 5) = x2iminus
Sheet1.Cells(x + 1, 5) = x2i

Next x
End Sub
```

For make this program we refer Application of Random vibration by N.C. Nigam ,
By use of this programming we get any number of random numbers white noise simulation and using this we also find power spectral density of the random process by taking Fourier transform.
For our simulation process shown in fig- 7

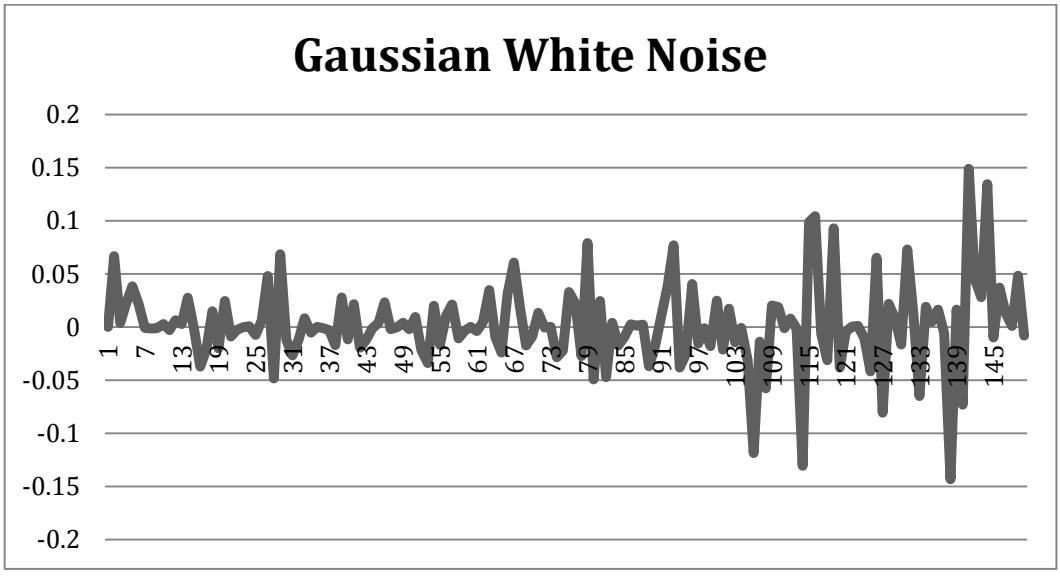

Figure 7 Gaussian White noise Simulation

3.2.3 Markov Process

A stochastic process X(t) is said to be a scalar Markov process if it has the property

$$\Pr\{X(t_n \leq x_n)|X(t_{n-1}) = x_{n-1}, \dots\dots, X(t_1) = x_1\} \tag{3.38}$$

$$\Pr\{X(t_n \leq x_n)|X(t_{n-1}) = x_{n-1}\} \tag{3.39}$$

Where Pr[-] denotes the probability of an event, and where the statement following a vertical specifies certain conditions under which such a probability is defined. In the present case, the conditions are known values of X(t) at earlier time instants, $t_1,\dots,t_{n-1}$.

A sufficient condition for X(t) to be Markov process is that its increment in any two non The conditional probability, $Prob\{X(t) \leq x|X(t_0) = x_0\}$ of a Markov process X(t) is called the transition probability distribution function. A Markov process is completely characterized by its probability distribution and its probability distribution at an initial time. If the transition probability distribution function of a Markov process is differentiable, its transition probability density function can be obtained, which is often easier to deal with. Transition probability density function can be obtained by:

$$q(x, t|x_0 t_0) = \frac{\partial}{\partial x} prob\{X(t) \leq x|X(t_0) = x_0\} \tag{3.40}$$

The concept of scalar Markov process is readily generalize to a vector Markov process. Thus X(t) $= \{X_1(t), X_2(t), \dots . Xm(t)\}'$ is an m dimensional Markov vector if it has the property of

$$Pr\left[\bigcap_{j=1}^{m}\{X_j(t_n) \leq x_j\}|X(t_{n-1}) = y_{n-1} \dots\dots X(t_1) = y_1\right] \tag{3.40}$$

$$Pr\left[\bigcap_{j=1}^{m}\{X_j(t_n) \leq x_j\}|X(t_{n-1}) = y_{n-1}\right] \qquad \text{tn >tn-1>….>t1} \tag{3.41}$$

Where ∩ denotes the joint occurrence of multiple events. A sufficient condition for vectorially valued stochastic process to be a Markov vector is that its vectorial increment be independent in non-overlapping time intervals. The transition probability density of a Vector Markov process is a generalization of

$$q(x,t|x_0,t_0) = \frac{\partial^m}{\partial_{x_1 \ldots\ldots \partial x_m}} prob\left[\cap_{j=1}^{m}\{X_j(t) \le x_j\}\,|X(t_0) = X_0\right] \qquad (3.42)$$

The higher order probability densities, describing the behaviour of a Markov process at several instants of time, can be constructed from initial probability density and transition probability density as follows;

$$Pr(x_1,t_1;x_2,t_2;\ldots\ldots;x_n,t_n) = q(x_n,t_n|x_n\text{-}1,t_n\text{-}1),q(x_n\text{-}1,t_n\text{-}1|x_n\text{-}2,t_n\text{-}2)\ldots q(x_2,t_2|x_1,t_1)p(x_1)$$
$$t_1<t_2<tn \qquad (3.43)$$

3.3 Exact Response Methods for Randomly Excited Systems

Mathematically exact solution of randomly excited nonlinear system is difficult to obtain. The possibility of solution does exist, however when random excitations are independent at any two instant of time, in which case the system response, represented as a vector in a state space, is Markov vector. The probability density of a Markov vector is governed by parabolic partial differential equation called the *Fokker-Planck* equation, which has been derived, in previous chapter.

Still the full solution to a Fokker-Planck equation, which shows how the probability structure evolves with time, know for very special first order systems for which system response is Markov scalar process.

For higher order non-linear systems, solution has been obtained in few cases for reduced Fokker-Planck equation without the time derivative term. The unknown in a reduced Fokker-Planck equation is the probability density of system response when it reaches to state of statistical stationary. Of course, a stationary response exists only if several necessary conditions are met, including:

(1) Every random excitation is stationary process,

(2) System parameters are time invariant, and

(3) Some energy dissipation mechanism exists in the system such that the energy input from the random excitations is balanced statistically by energy output from dissipation.

A stationary solution, if obtainable, is very useful. It is needed for computing the statistical averages for some system response variable to cross over specified boundaries, which are useful or reliability assessment. Stationary response also provides information on the possibility of instability and/or bifurcation of the response.

The class of nonlinear systems, for which this procedure is applicable, is termed the class of generalized stationary potential, and is claimed to be the broadest class of solvable, nonlinear, stochastic systems up to that date. The method is said to be also applicable to MDOF systems. However, the class of generalized stationary potential is too narrow with respect to the class of systems that we strive to cover.

3.4 Approximate Response Methods for Randomly Excited Systems

When a multidimensional nonlinear system is subjected to both parametric and external random excitations of Gaussian white noises, the reduced Fokker-Planck equation can be solved in closed form only with certain highly restrictive relation between the system parameters and the spectral densities of the excitations. Under Practical condition such restrictive requirement rarely met. Therefore approximate solution techniques are generally needed.

The approach to the problem of determining the time-dependent probability density function is to solve the FPK-equation by numerical means. A simple and efficient numerical scheme can be formulated by employing the random walk analogue Roberts [1981]. This method can only be applied to 1-dimensional FPK equations and is, therefore, not applicable to MDOF systems.

The numerical integration of the FPK-equation for MDOF systems quickly becomes very cumbersome, because of the high dimension of the probability space that is to be discretized. Summarizing, it can be concluded, that the Fokker-Planck equation method is not suitable to be applied to a wide class of practical MDOF systems. An even more important shortcoming of the method is that is does not provide information on the power spectral density of the response.

3.4.1 Perturbation Method

The basic appreciation is to expand the solution to the nonlinear set of equations in terms of a small scaling parameter, which characterizes the magnitude of the nonlinear terms in these equations. The first term in the expansion is simply the linear response, which is the response when all the nonlinearities in the system are removed. [10]
The subsequent terms express the influence of the nonlinearity. As with perturbation in general, the calculations are usually lengthy and rapidly become more tedious as the order of the scaling parameter increases. In practice, results are usually obtained only to the first order in the scaling parameter. The method is, therefore, only valid for small perturbations. Consequently, the perturbation method can only be applied effectively when weakly nonlinear systems are considered.
The Perturbation method can be extended to MDF systems. By expanding the response in powers of perturbation parameter, the equation can be reduced to hierarchy of a system of linear differential equation that can be solved sequentially by linear system theory.

3.4.2 The Fokker-Planck-Kolmogorov Equation Method

The Fokker-Planck (FPK) equation method can provide information on the stationary (or non-stationary) un-normalized probability density function of the response of a dynamic system. The FPK equation is a partial differential equation for the probability density function of the response incorporating partial derivatives to this response (and time, in case of a non- stationary probability density function). This equation should be solved under appropriate boundary (and initial) conditions. One could distinguish between methods providing exact solutions and methods providing approximate solutions of the FPK-equation.

3.4.3 Stochastic Averaging

In these methods the response of lightly damped systems to broad band excitation is approximated by a diffusion process. The coefficients of the associated FPK equation are derived based on an appropriate averaging of the equations of motion. The appeal of these methods lies in the fact that they often reduce the dimensionality of the problem and significantly simplify the solution procedures. On account of this advantage they are also applied to systems wherein the response is already Markov.
The methods of stochastic averaging enhance the scope of the FPK equation approach in random vibrations. The merit of these methods is that they lead to non-Gaussian estimates for the response.

3.4.4 Linearization Methods

This method is the most popular approach in nonlinear random vibration problems. It is extension of the well-known harmonic linearization technique to stochastic problems and is applicable to both SDOF and MDOF systems under stationary or non-stationary inputs. [11] The method consists of optimally approximating the nonlinearities in the given system by linear models so that the resulting equivalent system is amenable for solution. For evaluating the parameters in the equivalent system, an additional assumption that the response is Gaussian is generally made.

3.4.4.1 Stochastic Equivalent Linearization

The method of the stochastic equivalent linearization is based on the idea that a Nonlinear system may be replaced by a linear system by minimizing the mean square error of the two systems. This method has seen the broadest application because of their ability to accurately capture the response statistics over a wide range of response levels while maintaining relatively light computational burden. The method will be briefly discussed in the following sections.

3.4.4.2 Stochastic Equivalent Linearization Method

The equivalent linearization method has been studied widely. The development of an equivalent linearization method, reported in the literature, is by Booton [1953]. Caughey [1960] utilized a linearization technique for the response analysis of systems with a bilinear hysteresis restoring force.

3.4.4.3 System model

To illustrate the procedure of equivalent linearization theory [11], let us consider Following oscillator with a nonlinear restoring force component.

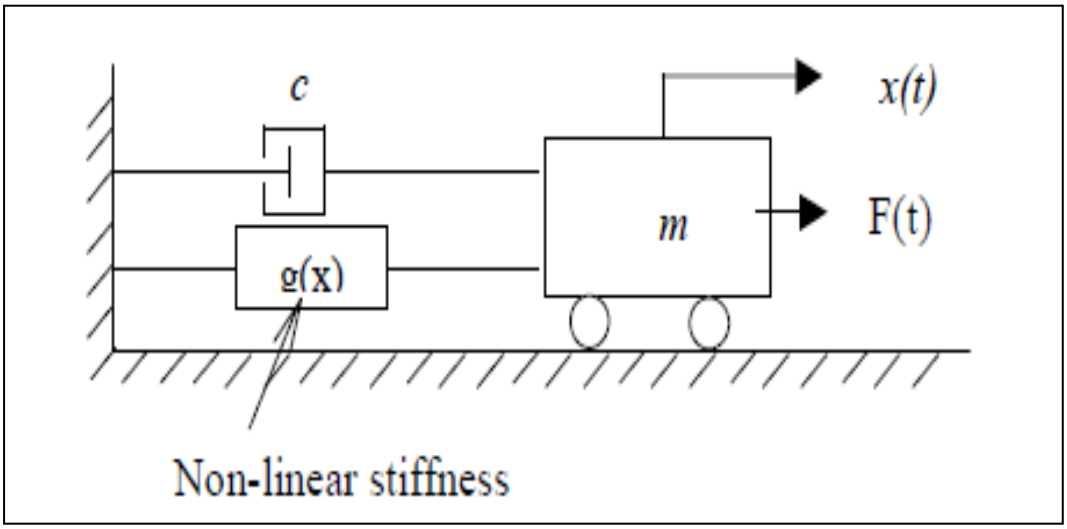

Figure 8 SDOF mass-spring-damper system

The ordinary differential equation of the motion can be written as:

$$m\ddot{x}(t)+c\dot{x}(t)+g(x)=F(t) \tag{3.44}$$

Where m is the mass, c is the viscous damping coefficient, $F(t)$ is the external excitation signal with zero mean and $x(t)$ is the displacement response of the system.
Dividing the equation by m , the equation of motion can be rewritten as:

$$\ddot{x}(t)+2\xi\omega_n\dot{x}(t)+h(x)=f(t) \quad (3.45)$$

where ω_n is the undamped natural frequency, for the linear system.
We can always find a way to decompose the nonlinear restoring to one linear component plus a nonlinear component [12].

$$h(x) = \omega_n^2(x + H(x)\lambda) \quad (3.46)$$

Where λ is the nonlinear factor to control the type and degree of nonlinearity in the system.

3.4.4.3 Implementation of the Stochastic Equivalent Linearization

The idea of linearization is replacing the equation (3.45) by a linear system:

$$\ddot{x}(t) + 2\xi_{eq}\omega_{eq}\dot{x}(t) + \omega_{eq}^2 x(t) = f(t) \quad (3.47)$$

ξ_{eq} is the damping ratio of equivalent linearized system and ω_{eq} is the natural frequency of the equivalent linearized system.
To find an expression for ω_{eq}, it is necessary to minimize the expected value of the difference between equations (3.45) and (3.47) in a least square sense. Now the difference is simply the difference between the nonlinear stiffness and linear stiffness terms, which is

$$e=h(x)- \omega_{eq}^2 x \quad (3.48)$$

The value of ω_{eq} can be obtained by minimizing the expectation of the square error $E\{e^2\}$.

$$\frac{d}{d\omega_{eq}^2} = E\{e^2\}=0 \quad (3.49)$$

Substituting the equation (3.48) into (3.49) performing the necessary ω_{eq} can be obtained as

$$\omega_{eq}^2 = \omega_n^2 \left(1 + \lambda \frac{E\{xH(x)\}}{\sigma_x^2}\right) \quad (3.50)$$

Where x σ is the standard deviation of *x(t).* This equation shows how the nonlinear component of the stiffness element affects the value of ω_{eq} .

In equation (3.50), the exact evaluation of $\lambda \frac{E\{xH(x)\}}{\sigma_x^2}$ requires knowledge of the first-order density function of the response process *x(t).* Let the process *x(t)* assume to be Gaussian, then the standard deviation can be found from equation (3.47). For convenience, the Duffing oscillator has been used to illustrate this procedure here, with which the nonlinear restoring force is written as:

$$h(x) = \omega_n^2 (x + x^3 \lambda) \quad (3.51)$$

Where the nonlinear factor λ controls the type and degree of nonlinearity in the system. A higher value of λ indicates a stronger nonlinearity. A positive value of λ represents a hardening system while a negative value represents a softening system behavior. In this case, the coefficient ω_{eq} can be expressed:

$$\omega_{eq}^2 = \omega_n^2 \left(1 + \lambda \frac{E\{x^4\}}{\sigma_x^2}\right) \quad (3.52)$$

Taking the density function of $x(t)$ to be of Gaussian form

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left\{-\frac{x^2}{2\sigma_x^2}\right\} \quad (3.53)$$

Using the definition of the expectation operator, it is found from equation (3.50) that:

$$\omega_{eq}^2 = \omega_n^2 (1 + 3\lambda A) \quad (3.54)$$

Where in this case

$$A = E[x^4] = \int_{-\infty}^{\infty} x^4 f_x(x) dx \quad (3.55)$$

By performing integration, the equation (3.54) can be expressed using Gamma function

$$A = \frac{2}{\sqrt{\pi}}\sigma_x^2 \Gamma\left(\frac{3}{2}\right) \tag{3.56}$$

Utilizing a well-known frequency domain input-output formula, the spectrum of x(t) be determined by

$$s_x(\omega) = |\alpha(\omega)|^2 s_f(\omega) \tag{3.57}$$

Where $s_x(\omega)$and $s_f(\omega)$are the spectral density matrix for x(t) and f(t) respectively.
Here the appropriate frequency response function $\alpha(\omega)$ is given as:

$$\alpha(\omega) = \frac{1}{(\omega_{eq}^2 - \omega^2 + 2f\xi_{eq}\omega_{eq}\omega)} \tag{3.58}$$

Then

$$\sigma_x^2 = \int_{-\infty}^{\infty} s_y(\omega) d\omega \tag{5.59}$$

Combining the equation (3.57) and 3.59), two algebraic relationships are obtained for the two unknown ω_{eq} and σ_x. By iterative process, finally, the desired equivalent coefficient can be obtained. The cyclic procedure can be devised as follow:

1. Assign an initial value to the equivalent coefficient ω_{eq}
2. Use equation (3.57), (3.58) and (3.59) to get σ_x
3. Solve equation (3.54) and 3.55) for the new ω_{eq}
4. Repeat steps 2 and 3 until results from cycle to cycle are similar.

3.4.5. Equivalent non linearization

This method is conceptually similar to the method of equivalent linearization and can be viewed as a generalization leading to non-Gaussian estimates for the response. It consists of replacing the given nonlinear system by an equivalent nonlinear system, which belongs to the class of problems, which can be solved exactly. This method is related to the class of exactly solvable FPK equations and thus is applicable only to systems under white noise inputs. The criterion of replacement is again the minimization of the mean square error. The method leads to non-Gaussian stationary response PDF and estimates

correctly the random response of limit cycle systems in which case, equivalent linearization fails.

Summary

Various methods for stochastic response analysis have been outlined in the previous sections. Exact solutions are obtainable from the FPK equation approach but are scarce. It is generally necessary to take recourse to one of the several approximate procedures available. Many of the approximations are based on the assumption that the response process is nearly Gaussian distributed and nearly Markovian in nature. The above methods have been applied in the past to study a variety of nonlinear problems such as structures undergoing large amplitude vibrations, yielding systems, self-excited systems, hysteretic systems and rocking of blocks. The developments of these methods are characterized by two conflicting objectives. Firstly, the methods are expected to be viable when applied to large-scale engineering structures, while; on the other hand, they need to capture correctly, the qualitative behavior of nonlinear systems. The linearization and closure methods are, perhaps, the only feasible analytical methods, which can be, used in conjunction with computational structural models for studying large-scale MDOF systems. One of the major drawbacks of these methods, however, lies in their inability to capture correctly the interactions between equilibrium states of the unforced system and external random excitations. It has been noted that, for systems under white noise excitations, the most probable response states correspond to the stable equilibrium states of the unforced system. Evidently, the linearization and closure techniques are ill equipped to model these nonlinear features satisfactorily. On the other hand, the averaging and FPK equation based approaches are mathematically well founded and perform well when applied to simple systems displaying the above mentioned complicated response patterns, but are, however, of limited use in analyzing large scale structures. Thus, methods to overcome these limitations still need to be developed. In preceding section we shows the method, which is overcome, the shortcoming of equivalent linearization method, Tail Equivalent linearization method.

Chapter 4 Brief Review of Reliability method

The aim of this section is to introduce the most common techniques of structural reliability analysis, namely, First Order Reliability Methods (FORM) and Second Order Reliability Method (SORM). First the concept of limit state equations and basic random variables is introduced. Thereafter the problem of error propagation is considered and it is shown that FORM provides a generalization of the classical solution to this problem. Different cases of limit state functions and probabilistic characteristics of basic random variables are then introduced with increasing generality [13].

4.1 Introduction

For structural components and systems first of all no relevant failure data are available, secondly failures occur significantly more rare and thirdly the mechanism behind failures is different. Structural failure occur not predominantly due to elderly processes but additionally due to the effect of severe events, such as extreme winds, snow fall, earth-quakes, or combinations. For the reliability assessment it is consequently necessary to consider the influences acting from the outside i.e. loads and influences acting from the inside i.e. resistances individually. It is thus necessary to establish probabilistic models for loads and resistances including all available information about the statistical characteristics of the parameters influencing these. Such information is such as data regarding the earth-quakes, experiment results of concrete compression strength, etc.
Determine the probability of failure establish probabilistic models for loads and resistances of a structural component. Probabilistic modeling is a two fold problem :
-Develop model for random variables,
-Study function of these random variables.
We will focus on the validity of the procedures rather than the validity of the results. If we have valid procedure and reasonable inputs, then our estimated probability will be

useful. We start by assuming an idealized structural component. The component has only two performance states,

-Safe

-Failure

All uncertainties can be quantified by random variable. We begin by considering a component with resistance R and load S, Probabilistic modeling of structural component is given in Fig.9, If S>R, component is failed and if S≤R component is safe.

Figure 9 Probabilistic modeling of structure

For a structural component for which the uncertain resistance R may be modeled by a random variable with probability density function f_R (r) subjected to the load S the probability of failure may be determined by

$$P_f = P(R \leq S) = F_R(S) = P(R|S \leq 0 \quad (4.1)$$

In case also the load is uncertain and modeled by the random variable S with probability density function $f_S(s)$ then probability of failure is

$$P_f = P(R \leq S) = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(s) f_s(s) dx \quad (4.2)$$

where $F_R(s) = CDF\ for\ R$ and $f_s(s)$ = PDF of S

Assuming that the load and the resistance variables are statistically independent. This case is called the fundamental case. The integration in Equation (4.2) is illustrated by convolution integral in Figure 10.

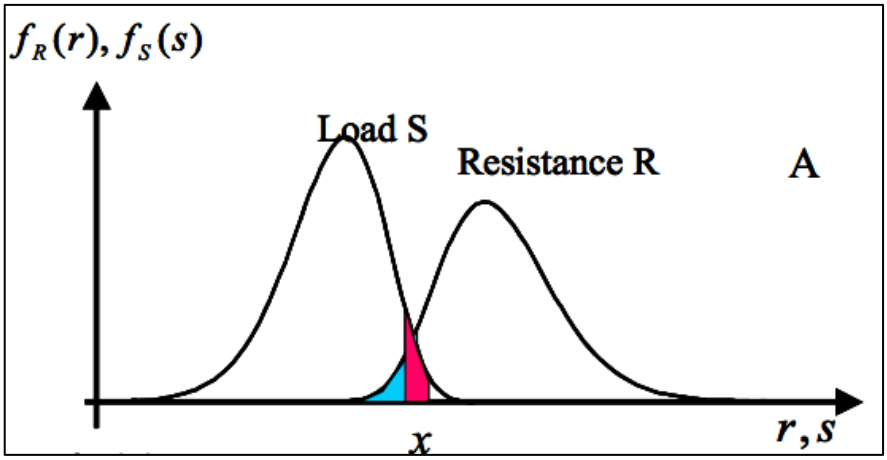

Figure 10 Illustration of the integration of Eq 4.2

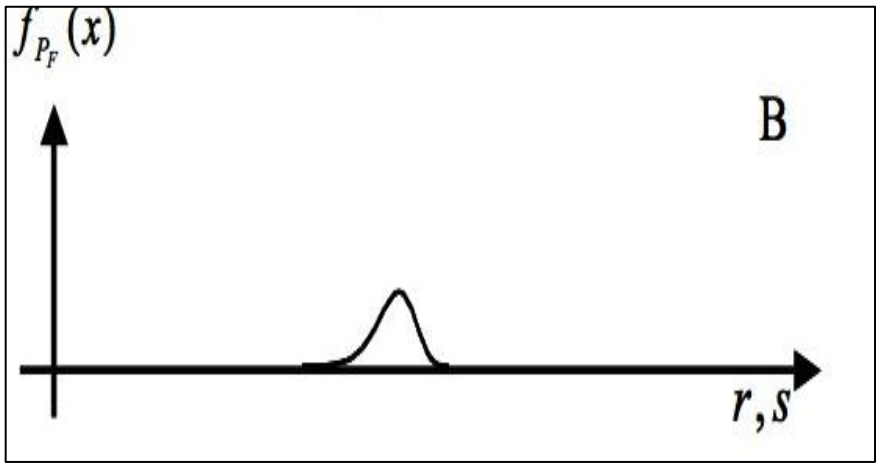

Figure 11 The distribution of the failure probability over the realization of R and S

This collared area shown the probability of failure,

$$P_f = F_R(s)f_s(s)$$

In Fig. 10 the contributions to the probability integral in Equation (4.2) are illustrated. Note that the probability of failure is not determined through the overlap of the two curves. In Fig.11, the density function for the failure probability is illustrated as a

function of the realizations of the random variables R and S. The integral under this density is not equal to 1 but equal to the failure probability P_f. Finally from Fig. 11, the most likely combination of values of r and s leading to failures may be identified by the mode of the density function for the probability of failure. This point is also often referred to as the design point r^*, s^*

There exists no general closed form solution to the integral in Equation (I) but for a number of special cases solutions may be derived. One case is when both the resistance variable R and the load variable S are normally distributed. In this case the failure probability may be assessed directly by considering the random variable M often referred to as the safety margin,

$$M = R - S$$

Then whereby the probability of failure may be assessed through

$$P_F = P(R - S \leq 0) = P(M \leq 0) \tag{4.3}$$

where M is also being normal distributed with parameters

$$\mu_M = \mu_R - \mu_S$$

$$\sigma_M = \sqrt{\sigma_R^2 - \sigma_S^2} \tag{4.4}$$

The failure probability may now be determined by use of the standard normal distribution function as

$$P_F = \phi\left(\frac{0-\mu_M}{\sigma_M}\right) = \phi(-\beta) \tag{4.5}$$

Where $\frac{\mu_M}{\sigma_M} = \beta$ is called safety index.

The geometrical interpretation of the safety index is illustrated in Figure 12.

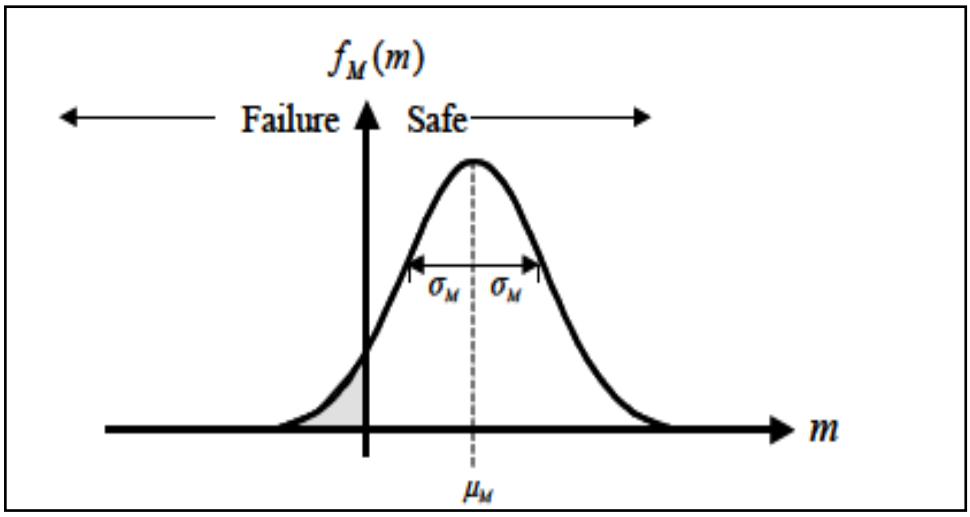

Figure 12 Illustration of the probability density function for normally distributed safety and margin

From Figure 12 it is seen that the safety index may be interpreted as the number of standard deviation by which the mean value of the safety margin.

In the general case the resistance and the load cannot be described by only two random variables but rather by functions of random variables.

$$R = f_1(x)$$

$$S = f_2(x) \tag{4.6}$$

where X is a vector with n so-called basic random variables. As indicated in Equation (4.6) both the resistance and the loading may be a function of the same random variables and R and S may thus be statistically dependent. Furthermore the safety margin

$$M = R - S = f_1(x) - f_2(x) = g\,(\mathrm{X}) \tag{4.7}$$

The function $g(\mathbf{x})$ is usually denoted the limit state function, i.e. an indicator of the state of the considered component. For realizations of the basic random variables X for which $g(\mathbf{x}) \leq 0$ the component is in a state of failure and otherwise for $g(\mathbf{x}) > 0$ the component is in a safe state.

Setting $g(\mathbf{x}) = 0$ defines a (n-1) dimensional hyper surface in the space spanned by the n basic random variables. This hyper surface is denoted the failure surface and thus separates all possible realizations of the basic random variables X resulting in failure, i.e. the failure domain, from the realizations resulting in a safe state, the safe domain.

Reliability in general

Structure reliability problem is also define in terms of a set of basis random variable.
The methods of structural reliability aim at computing the probability of failure of a structural system. The structural reliability problem is defined in terms of a set of basic random variables $x=(x_1,x_2\ldots.x_n)$ that describe the uncertain quantities. Affecting the state of the structure, and a set of m limit-state functions $g_k(x,\theta)$, k=1,2,…m, that describe the failure event of interest, where $\theta = (\theta_1, \ldots. \theta_p)$ denotes a set of p deterministic parameters. the structural system is seen as composed of components, and the k-th limit-state functions is defined such that $\{g_k(x, \theta) \leq 0\}$ denotes the events of failure of the k-th component in the outcome space of the random variable x. the probability of failure of the system,denoted p_f , is given by

$$P_f=\int_{\emptyset} f(x)dx \tag{4.8}$$

Where f(x) denotes the joint probability density function of x, and ∅ is the failure domain defined in terms of the limit-state functions $g_k(x,\theta)$, k=1,2,….m. depending on how the failure domain is defined, the structural reliability problem falls into one of the following four categories:
A structural "component" reliability problem is one in which the failure domain is defined by a single limit-state function, i.e.

$$\emptyset = \{g(x, \theta) \leq o\}$$

A structural "series system" reliability problem is one in which the failure domain is defined as the union of component failure events, i.e.

$$\emptyset = \left\{\bigcup_{k=1}^{m} g_k\,(x, \theta) \leq o\right\}$$

Where m is the number of components in the series system. Thus, a series system fails if any of its components fail.

A structural "parallel system" reliability problem is one in which the failure domain is defined as the intersection of component failure events, i.e.

$$\emptyset = \left\{ \bigcap_{k=1}^{\infty} g_k(x, \theta) \leq 0 \right\}$$

Where m is the number of components in the parallel system. Thus a parallel system fails if every one of its components fails.

A structural "general system" reliability problem is one in which the failure domain is defined in terms of both unions and intersections of components failure events. In general, such system can be formulated either in terms of a series system of parallel subsystem, or in terms of a parallel system of series subsystems. The former formulation leads to the definition

$$\emptyset = \left\{ \bigcup_{k} \bigcap_{k \in c_k}^{\infty} g_k(x, \theta) \leq 0 \right\}$$

Where c_k is the k-th set which is the set of any subset of component which joint failure constituents failure of the system.

For most structural reliability problems, an analytical evaluation of the integral is impossible and, therefore, numerical methods have been developed for approximate analysis. The first and second- order reliability methods, commonly known as FORM and SORM, are widely used as approximate solution tools for such analysis. In the following section, these methods and their related topics are briefly described. More detailed development of these methods can be found in Ditlevsen and Madsen [14].

4.2 First Order Reliability Methods and second order reliability method

4.2.1 Introduction

It is one of the most important methods for reliability evaluations in structural reliability theory. Several commercial computer codes have been developed for FORM analysis and the methods are widely used in practical engineering problems and for code calibration purposes. In this section first the basic idea behind the FORM methods will be highlighted and thereafter the individual steps of the methods will be explained in detail [6].

4.2.2 Failure Events and Basic Random Variables

In reliability analysis of technical systems and components the main problem is to evaluate the probability of failure corresponding to a specified reference period. However, also other non-failure states of the considered component or system may be of interest, such as excessive damage, unavailability, etc. In general any state, which may be associated with consequences in terms of costs, loss of lives and impact to the environment are of interest. In the following we will not different between these different types of states but for simplicity refer to all these as being failure events, however, bearing in mind that also non-failure states may be considered in the same manner.

It is convenient to describe failure events in terms of functional relations, which if they are fulfilled define that the considered event will occur. A failure event may be described by a functional relation, the limit state function $g(\mathbf{x})$.

In Equation (4.8) the failure event **F** is simply defined as the set of realization of the function $g(\mathbf{x})$, which are zero or negative.

As already mentioned other events as failure may be of interest in reliability analysis and e.g. in reliability updating problems also events of the following form are highly relevant.

$$\mathrm{I} = \{ h(\mathrm{x}) = 0 \}$$

$$\boldsymbol{P_F} = \int_{\boldsymbol{g(x)\leq 0}} \int \boldsymbol{f_x(x)}\, \boldsymbol{dx} \tag{4.7}$$

where (**x**) **X** f is the joint probability density function of the random variables **X** . This integral is, however, non-trivial to solve and numerical approximations are expedient. Various methods for the solution of the integral in Equation (4.7) have been proposed including numerical integration techniques, Monte Carlo simulation and asymptotic Laplace expansions. Numerical integration techniques very rapidly become in efficient for increasing dimension of the vector **X** and are in general irrelevant. Monte Carlo simulation techniques may be efficient but in the following we will direct the focus on the widely applied and quite efficient FORM methods, which furthermore can be shown to be consistent with the solutions obtained by asymptotic Laplace integral expansions.

FORM and SORM are approximate methods for computing the probability integral (). In these methods, the limit-state surface $g_k(x,\theta) = 0$ for each component is replaced by a first or second order approximating surface, respectively. The approximation is carried out at a point, known as the design point, which is the point on the surface $g_k(x,\theta) = 0$ that is nearest to the origin in a transformed standard normal space.

FORM approximates the limit-state surface of each component as a tangential hyperplane at the design point. For a component reliability problem, the accuracy of FORM is primarily dictated by the nonlinearity of the limit-state surface around the design point. This is because the dominant contribution to the probability intergral comes from the neighborhood of this point, where the probability density achieves its maximum value. On the other hand, SORM approximation the Limit-state surface as a hyperboloid, which can reflect the nonlinearity around the design point up to the second order. For nonlinear problems, SORM usually gives a better approximation than FORM.

For FORM or SORM analysis, we transform the random variables x into standard normal random variables u through a suitable one-to-one mapping T:x→ u. The limit-state functions $g_k(x,\theta)$, k=1,2,….m, are accordingly transformed such that $G_k(u,\theta) =$

$g_k[T^{-1}(u), \theta]$. The design point u_k^*, k=1,2,…m, are obtained by solving the constrained optimization problems.

$$u_k^* = argmin\{\|u\| : u \in \emptyset(u, \theta) = 0\} \quad (4.8)$$

After finding the design points, we replace the limit-state surfaces $G_k(u, \theta) = 0$, k=1,2,….m, by the first order (in FORM) or second (in SORM) approximating surfaces at the design points.

In FORM, the k-th approximating surface is a hyperplane defined by

$$G_k(u, \theta) \equiv \nabla_u G_k^T(u - u_k^*) = \|\nabla_u G_k\|(\beta_k - \alpha_k^T u) \quad (4.9)$$

Where $\nabla_u G_k$ denotes the gradient vector of $G_k(u, \theta) = 0$ with respect to u evaluated at the design point u_k^*,$\alpha_i = -\nabla_u G_k / \|\nabla_u G_k\|$ is the normalized negative gradient vector, and β_k is the distance of the hyper-plane from the origin. For the component reliability problem in (), the first order approximation of p_f is given by

$$p_f \equiv \emptyset(-\beta) \quad (4.10)$$

Where β is the reliability index obtained above and $\emptyset(.)$ is the standard normal distribution function. For the series and parallel system problems, the first order solutions are given by

$$p_{fs} \cong 1 - \emptyset_m(\beta, R)$$
$$p_{fp} \cong \emptyset_m(\beta, R) \quad (4.11)$$

Where subscripts s and p respectively refer to the "series" and " parallel" system, and $\emptyset_m(\beta, R)$ denotes the m- variable standard normal distribution with zero means, unit variances and correlation matrix R at the point β^T=[$\beta_{1,\dots}$ β_2] . the element i,j of the correlation matrix is given by $\rho_{ij} = \alpha_i^T \alpha_j$. For m=2, i.e. systems with two components, one can show that

$$\emptyset_2(\beta_1, \beta_{2,}\rho_{12}) = \emptyset(\beta_1)\emptyset(\beta_2) + \int_0^{\rho_{12}} \phi_2(\beta_1, \beta_{2,}\rho)d\rho \qquad (4.12)$$

Where $\emptyset_2(\beta_1, \beta_{2,}\rho)$ is the bi-variate standard normal probability density function with zero means, unit variances and correlation ρ.

In SORM, the surfaces are replaced by approximating parabolic surfaces fitted at the design points u_k^*,k=1,2…m. Due to the difficulty in incorporating the multiple

A parabolic surface, however, SORM is usually used for component problems only. For a component reliability problem, Der kiureghian [8]has suggested the following formula

$$p_f \cong \phi(-\beta)\prod_{i=1}^{n-1}\frac{1}{\sqrt{1+\beta k_i}} \qquad (4.13)$$

Where k_i denote the principal curvatures of the limit-state surface at the design point, taken positive when the surface curves away from the origin near the design point. More accurate formula for the SORM approximation have been suggested subsequently.

4.2.3 Linear Limit State Functions and Normal Distributed Variables

For illustrative purposes we will first consider the case where the limit state function $g(\mathbf{x})$ is a linear function of the basic random variables X . Then we may write the limit state function as

$$g(x) = a_0 + \sum_{i=1}^{n} a_i x_i \qquad (4.14)$$

If this basic random variables are normally distributed we furthermore have that the linear safety margin M defined through

$$M = a_0 + \sum_{i=1}^{n} a_i X_i \qquad (4.10)$$

It is also normally distributed with mean value and variance

$$\mu_M = a_0 + \sum_{i=1}^{n} a_i \mu_{x_i} \qquad (4.11)$$

$$\sigma_M^2 = \sum_{i=1}^{n} a_i^2 \sigma_{x_i}^2 + \sum_{i=1}^{n} \rho_o a_i a_j \sigma_i \sigma_j \quad (4.12)$$

where ρ_{ij} are the correlation coefficients between the variables X_i and X_j .

Defining the failure event by Equation (4.8) we can write the probability of failure as

$$P_F = P(g(x) \leq 0) = P(M \leq 0) \quad (4.13)$$

which in this simple case reduces to the evaluation of the standard normal distribution function

$$P_F = \phi(-\beta) \quad (4.14)$$

Where $\beta = \frac{\mu_M}{\sigma_M}$ is called reliability index.

Then the reliability index has the simple geometrical interpretation as the smallest distance from the line (or generally the hyper-plane) forming the boundary between the safe domain and the failure domain, i.e. the domain defined by the failure event. It should be noted that this definition of the reliability index does not depend on the limit state function but rather the boundary between the safe domain and the failure domain. The point on the failure surface with the smallest distance to origin is commonly denoted the design point.

It is seen that the evaluation of the probability of failure in this simple case reduces to some simple evaluations in terms of mean values and standard deviations of the basic random variables, i.e. the first and second order information.

CHAPTER 5 Tail Equivalent Linearization Method

5.1 Introduction

Tail equivalent linearization method is based on first order reliability method, which obtains an equivalent linear system for the given nonlinear problem with equal tail probability related to a specified time and threshold. In TELM, the input process is discretized and represented by a set of standard normal random variables. Each response threshold defines a limit state surface with the "design point" being the point on the surface that is nearest to the origin. Linearization of the limit-state surface at this point uniquely and non-parametrically defines a linear system, denoted as Tail-Equivalent Linear System (TELS). There is no need to characterize the linear system in terms of its order, degrees of freedom, or parameters. The tail probability of the TELS response for the specified threshold is equal to the first-order approximation of the tail probability of the nonlinear system response for the same threshold. For this reason, the name Tail-Equivalent Linearization Method (TELM) is used. Once the TELS is defined for a specific response threshold, methods of linear random vibration analysis are used to compute various response statistics, such as the CDF and PDF at a given time, the mean crossing rate, and the distribution of the maximum response over an interval and tail probabilities of local and extreme peaks. The method has been developed for application in both time, Fujimura and Der Kiureghian (2007), and frequency domain, Garrè and Der Kiureghian (2010), and it has been applied for inelastic structures as well as structures experiencing geometric nonlinearities.

5.2 Representation of Input Excitations

The essential step in the application of FORM or TELM is time-based discretizing of the excitation in terms of standard normal random variables. There are various method

available for discrete representation of random processes, including a method that is particularly appropriate for modeling earthquake ground motions.

For a Gaussian process with a mean function $\mu(t)$, virtually all-existing discrete representation methods lead to the form:

$$F(t) = \mu(t) + \sum_{i=1}^{n} S_i(t) U_i = \mu(t) + S(t)U \quad (5.1)$$

where $U = [u_1, u_2 \ldots\ldots u_n]^T$ is a vector of standard normal variables,

$S(t)=[s_1(t), s_2(t) \ldots\ldots s_n(t)]^T$ is a vector of deterministic basis functions dependent on the covariance structure of the process, and n is a measure of the resolution of the representation. [6]

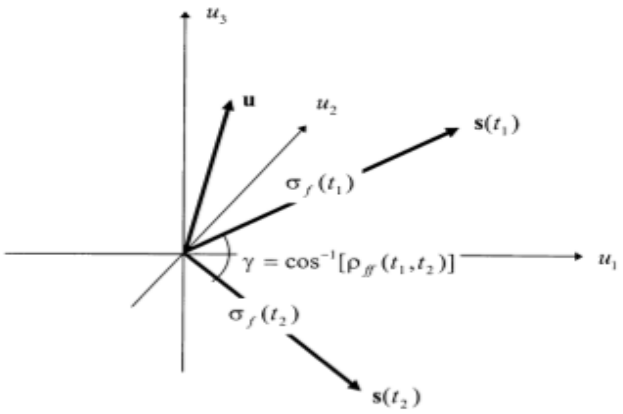

Figure 13 Geometric representation of a Gaussian process in standard normal space

The main difference between various representation methods lies in the selection of the basis functions. Some of them are given below

1. .Karhunen-Loeve expansion method

$$S_i(t) = \sqrt{\lambda} i \psi i(t) \quad (5.2)$$

where λ_I ith eigenvalue and ψ_I ith eigenfunction associated with the covariance function of the process.

2. .Representation using trigonometric polynomials

$$S(t) = \propto_n(t)^T L \tag{5.3}$$

Where $\propto_n(t)$ a vector of simple trigonmentric functions and L is a lower triangular matrix related to the covariance function of the process.

3.Expansion Optimal Linear Estimation (EOLE) method

$$s_i(t) = \psi_i^T \sum_{f(t)} F/\sqrt{\lambda_i} \tag{5.4}$$

Where λ_I ith eigenvalue and ψ_I ith eigenfunction associated with the covariance matrix of an n-vector F containing the values of f(t) at selecting points t_i, i = 1,2,…n, and $\sum_{f(t)} F$ denotes the vector of f(t) with F.

4**) The orthogonal series expansion method**

It is similar to the karhunen – Loeve expansion but employs a set of orthogonal function that are not necessarily eigenfunctions. The basis function is represented by $S_i(t)=\sqrt{\lambda}\,\phi_i(t)$, where λ_i and $\phi_i(t)$ are computed from orthogonal functions and the covariance function of the process.

Another method that is of particular interest in earthquake engineering represents the process in terms of the response of a filter to a train of random pulses. (The pulses may represent intermittent ruptures at the fault, whereas the filter may represent the medium through which the waves travel.). If the pulses are Gaussian and the filter is linear, the process *f*(*t*) is Gaussian. This representation can also be written in form of (5.1). The random variables *ui* then represent the pulse magnitudes at discrete times *ti*, *i* = 1,…,*n*, whereas *Si*(*t*) = *s*(t-t_i) where *s*(*t*) denotes the unit-impulseresponse function of the filter. This Processes with different characteristics can be modeled by properly selecting the filter. [8]

It is evident from equation (5.1) that all discrete representation method expressed as the Gaussian process as the scalar product of two vectors.

A time variant deterministic vector s(t), that is the basis functions Which describe the time evaluation of the process.

A time invariant vector u, that is standard random variable which describe the randomness of the excitation.

For a zero mean ($\mu(t) = 0$), second order i.e. finite variance ($\sigma^2 = 1$) for that Gaussian process representation has;

$$f(t) = \sum_{i=1}^{n} \text{ui si(t)} = s(t)^T u \tag{5.5}$$

Where $u = [u_1, u_2, \ldots\ldots\ldots\ldots\ldots u_n]^T$ is a vector of standard normal variables, $s(t) = [s_1(t), s_2(t), \ldots\ldots, S_n(t)]^T$ is a vector of deterministic basis functions dependent on the covariance structure of the process, and n is a measure of the resolution of the representation the main difference between various representation methods lies in the selection of the basis functions when the random process is represented in (5.5 [1]).

Here we use time basis function in time domain formulation that is smooth version of filtered pulse train described in Der-kiureghin is used. Suppose that the process f(t) is the response of the linear filter to a white noise excitation W(t).

Let the process f(t) we described as the output of a linear filter excite a white noise W(t)

$$f(t) = \int_0^t W(\tau) h(t - \tau)\, d\tau \tag{5.6}$$

For a stable and finite variance in response to white noise, the process f(t) becomes stationary after a duration at which the IRF $h_f(t)$ diminishes to zero. In place of discretizing f(t) directly we can discretize w(t), that is white noise.

Consider the sequence of equally spaced time points $t_i = t_i{-}1 + \Delta t$, i = 1, 2, . . . , t_n, with $t_0 = 0$ and Δt a small time step. For discretizing white noise W(t) we use approximately rectangular wave process defined by;

$$\acute{W}(t) = \frac{1}{\Delta t} \int_{t_{i-1}}^{t_i} w(\tau) d\tau \qquad t_{i-1} < t \leq t_i \tag{5.7}$$

It is easy to show that the wave amplitude

$$w_i = \acute{W}(t), \qquad t_{i-1} < t < t_i$$

$$w_i = \frac{1}{\Delta t}\int_{t_{i-1}}^{t_i} w(\tau)d\tau \tag{5.8}$$

One can show that w_i are statistically independent Gaussian random variables with zero mean and constant variance, $\boldsymbol{\sigma^2 = 2\pi S\Delta t}$ where S is the intensity of the white noise. The sequence w_i, i=1,2…n represents W(t) in the discrete form of a random pulse train. The response of the linear filter to this pulse train is also of the from eq- (5.5) with the basis function $s_i(t)$ which is defined is below. We introduce the standard normal variable :

$u_i = w_i/\sigma$

From (5.5) $$f(t) = \sum_{i=1}^{n} u_i s_i(t) = \sum_{i=1}^{n} \frac{w_i}{\sigma} s_i(t) \tag{5.10}$$

From (5.6) $$f(t) = \int_0^t W(\tau)h(t-\tau)\,d\tau = \int_0^t w_i h(t-\tau)\,d\tau \tag{5.11}$$

From equation (5.10) and (5.11)

$$S_i(t) = \sigma \int_{t_{i-1}}^{t} h_f(t-\tau)\,d\tau \qquad t_{i-1} < t \leq t_n$$

$$= 0 \quad \text{otherwise} \tag{5.12}$$

where $h_f(t)$ denotes the impulse-response function (IRF) of the filter.

For understanding the importance of $h_f(t)$ see example.1

Example 1.Consider a second order filter subjected to a pulse train defined by

$$\ddot{x_f}(t) + 2\xi_f\omega_f\dot{x_f}(t) + \omega_f^2 x_f(t) = f(t) \tag{5.13}$$

The impulse response function of the given above system for f(t) is given as below

$$h_f(t) = e^{(-\xi_f\omega_f t)}\left[\left(\frac{(2{\xi_f}^2-1)\omega_f}{\sqrt{1-{\xi_f}^2}}\ \sin\left(\omega_f\sqrt{1-{\xi_f}^2}t\right)\right) - 2\xi_f\omega_f\cos\left(\omega_f\sqrt{1-{\xi_f}^2}t\right)\right] \tag{5.14}$$

The above characterization defined as a stationary process earthquake motions, however are typically non- stationary in both time and frequency domains [8].

To account for temporal, we multiply the pulse train by a deterministic modulating function q(t). The discrete representation in become eq. (5.5)

$$F(t)=q(t)\ \{\mu(t) + \sum_{i=1}^{n} u_i s_i(t)\} \tag{5.15}$$

For typical earthquakes, q(t)) tends to gradually increase to a constant plateau and then decrease to zero at termination of the motion. To account for spectral non-stationarity, one can change the filter properties with time. However a simpler approach is to use multiple filters each with its own modulating function. Hence we write the most general discrete form of the input process as

$$F(t) = \sum_k q_k(t)\ \{\mu(t) + \sum_{i=1}^{n} u_i s_i(t)\} \tag{5.16}$$

Where q_k (t) denotes the modulating function for the k-th filter and s_{ik} denotes the basis function obtained from for the k-th filter. By proper selection of filter properties and modulating functions, almost any kind of temporal and spectral non- stationary can be modeled. Figure (5.2) shows an example of the non-stationary excitation generated using Eq(5.12) & Figure (5.2)(a) shows the time modulation functions for two filters, whose

system properties are $\omega_{f1} = 5\pi rad/sec$ and $\xi_{f1} = 0.4$ for the first filter and $\omega_{f2} = 2\pi\, rad/sec$ and $\xi_{f2} = 0.2$ for the second filter. Fig –(13) shows a sample of a train of pulse with $\Delta t = 0.02$ sec approximating the white noise with the intensity S=1(m/sec^2)^2. We have used for the impulse response function of the two filters to generate the non-stationary excitation in Figure (15). This resembles the actual pattern of earthquake accelerograms.

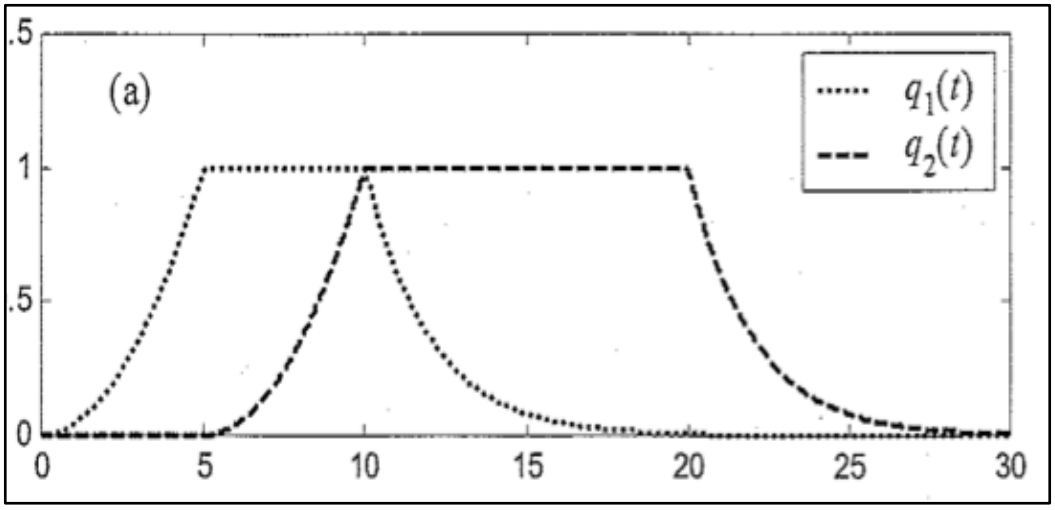

Figure 14 modulation functions q(t)

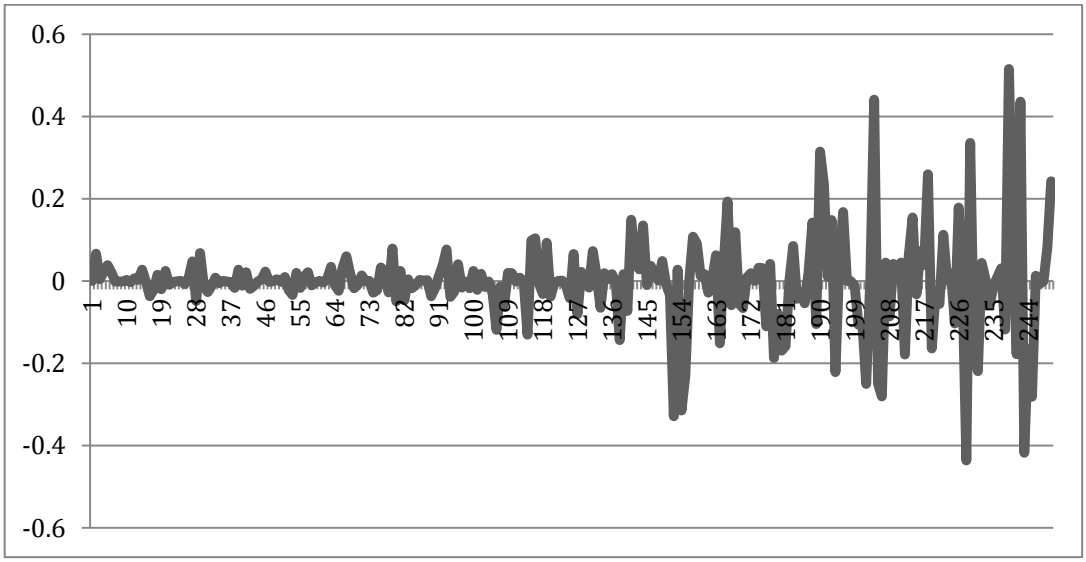

Figure 15 Random sample of white noise

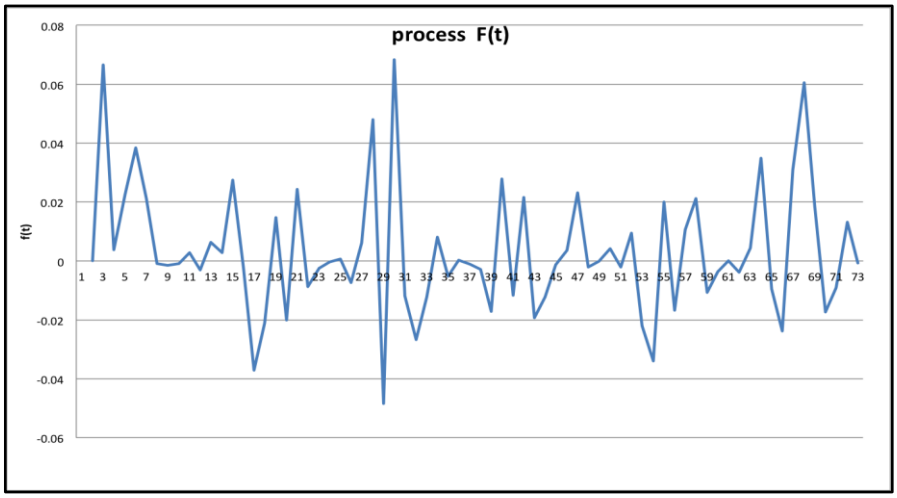

Figure 16 Random sample excitation f(t)

For non- Gaussian processes, representation by a linear relation shown in equation (5.5) is not possible. The non- Gaussian process is defined as a nonlinear function of the vector u. Several methods of representation in terms of standard normal variables are available, depending on the manner in which the process is defined. If the process is defined in terms of a set of non- Gaussian random variables, then the Rosenblatt transformation can be used to transform these variable into standard normal variables. If the process is defined by a nonlinear translation of a Gaussian process then the underlying Gaussian process can be represented in the form of equation (5.5) example of discretized non- Gaussian processes can be found in given below example;

Example- 2 Consider the response of a linear oscillator described by the differential equation

$$\ddot{x} + 2\xi_n\omega_n\dot{x} + \omega_n^2 x = f(t) \qquad (5.17)$$

Where where ω_n is the natural frequency and ξ_n the damping ratioof the oscillator. The unit-impulse-response function of the system is given below

$$h_f(t) =$$

$$\exp(-\xi_n\omega_n t)\left[\left(\frac{(2{\xi_n}^2-1)\omega_f}{\sqrt{1-{\xi_n}^2}}\ \sin\left(\omega_n\sqrt{1-{\xi_n}^2}t\right)\right)-2\xi_n\omega_n\cos\left(\omega_n\sqrt{1-{\xi_n}^2}t\right)\right]$$

In the following analysis $\omega_n = 4\pi$ rad/sec & $\xi_n = 0.05$ are used

Let y(t) be a Gaussian process with zero mean and unit variance. We consider three excitation process f(t) defined by

Process 1 f(t)=y(t)

Process 2 f(t)= y(t)$|y(t)|^{\theta}/\sqrt{E[|y(t)|^{2+2\theta}]}$

Process 3 f(t)= $\exp[\lambda + \xi y(t)] - m$;

where 0< m, $\xi = \sqrt{\ln(1+\frac{1}{m^2})}$ and $\lambda = \ln m - 0.5\xi^2$

All the three processes have zero mean and unit variance. Process 1 represents a Gaussian excitation, whereas processes 2 and 3 represent non-Gaussian excitations obtained by translations of y(t). One can easily verify that the latter two processes approach the Gaussian process as $\theta = 0$ and m = 0. Hence, the parameters θ and 1/m describe the degree of non-Gaussianity of f(t). It is noted that process 3 is a shifted lognormal process. The first-order probability density functions of the three processes for θ =1 and m=1 are shown in Fig. 3. Note that process 2 has a symmetric distribution with infinite density at the zero point, whereas process 3 has an asymmetric distribution and is bounded from below.

The Gaussian process *y*(*t*) is represented in the discretized form of Eq. (1) by filtering a train of random pulses of magnitude u_i, i=1,2,…, with normalized basis functions

$S_i(t) = \sigma \int_{t_{i-1}}^{t} h_f(t-\tau)\ d\tau\ \ t_{i-1} < t \leq t_n$

where t_i are equally spaced time points with t_1=0.

Sample realizations of the three processes for θ =1 and m= 1 are shown in Fig. 4. All three processes are asymptotically stationary for large t.

Although the focus of our study is on stationary process. The discretization in (5.7) and (7.8) effectively cuts off the frequency content of the white-noise process beyond a frequency roughly equal to $\pi/\Delta t$ rad/s. Therefore, care should be exercised in the selection of the discretization time step Δt . If, for the system of interest, the upper limit of significant input frequencies is ω_{max}, then $\Delta t < \pi / \omega_{max}$, must be selected.

5.3 Characterization of linear system

In this section we examine the response of a general linear, time-invariant system to a stochastic excitation defined in the form of (5.5) using the time-domain discretization scheme just described. Consideration of the system response in the space of the standard normal random variables u leads to a geometric characterization of the system and an inverse relation for its IRF. In other word we say that the system having stochastic excitation given in form of (5.5), For such system the response can be obtain by Duhamel's integral at time t_x. For simplicity we write (5.5) in following form [6]:

$$F_j(t) = \sum_{i=1}^{n} U_j^i \; S_j^i(t) = S_j(t \tag{5.18}$$

It's the component of the excitation in the j^{th} direction at discrete time point t = t_0,t_1…t_n where t_i = i * $t_i = i \, . \, \Delta t$.

U_j = $\{U_j^1, U_j^2, \ldots U_j^n\}^T$ = vector of standard normal random variable and

$S_j(t)$ = $\{S_j^1(t), \ldots S_j^n(t)\}$ = basis function in the j=1,2,….m direction.

The desired response X(t) which is affected by base excitation can be written as follows:

$$X(t) = \sum_{j=1}^{m} \int_0^{t_x} F_j(\tau) \, h_j(t_x - \tau) \, d\tau$$

(5.19)

where $h_j(t)$ is the IRF in the jth direction. By substituting Eq. (5.18) in (5.19):

$$X(t_x,u) = \sum_{j=1}^{m} \int_0^{t_x} \sum_{i=1}^{n} U_j^i \; S_j^i(\tau) \; h_j(t_x-\tau)\, d\tau$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{n} U_j^i \; a_j^i(t) \; = \sum_{j=1}^{m} a_j(t) \; U_j \tag{5.20}$$

where

$\boldsymbol{a_j^i(t)} = \int_0^t \boldsymbol{S_j^i(\tau)} \; \boldsymbol{h_j(t_x-\tau)d\tau}$

& $$\boldsymbol{a_j} = \{a_j^1, a_j^2, \ldots\ldots a_j^n\} \tag{5.21}$$

With the definitio $a(t) = [a_1(t), a_2(t), \ldots . a_m(t)]^T$ is a row vector collecting the n Duhamel integrals of the deterministic basis function, the above equation can be written as

$$X(t,u) = a(t)\, U$$

and this means that for linear systems the response can be stated as the product of two vectors which one of them is deterministic time variant [S(t)] and the other is random time invariant (U), if the excitation is stated as the product of two vectors like Eq. (5.5). The geometric interpretations made earlier for the Gaussian process F(t) apply to the response process X(t) as well. In addition, we note that the cross-correlation coefficient between the response $X(t_1)$ and excitation $F(t_2)$ is identical to the cosine of the angle between the respective basis function vectors $a(t_1)$ and $s(t_2)$ from fig 5.1.

Note: Dot product or scalar product [15] of two vector is

$\bar{A}.\bar{B} = |\bar{A}||\bar{B}| \cos\theta$

Consider the set of realizations of $F(t)$ that give rise to the event $\{X(t_0) \geq x_0\}$ at time $t_x = t_0$, where x_0 is a selected threshold. These correspond to realizations of **U** that satisfy the condition

$$x_0 - A(t_0)^T U \leq 0 \tag{5.22}$$

In the space of **U**, these lie in a half space bounded by the hyper-plane

$$x_0 - A(t_0)^T U = 0$$

having the unit normal

$$\widehat{\propto (t_0)} = A(t_0)/\|A(t_0)\|$$

and distance $\beta(x_0, t_0) = x_0/\|A(t_0)\|$ from the origin. This is illustrated in Fig. 17 in the plane formed by the coordinate u_1 and the vector $\widehat{\propto (t\_0)}$.

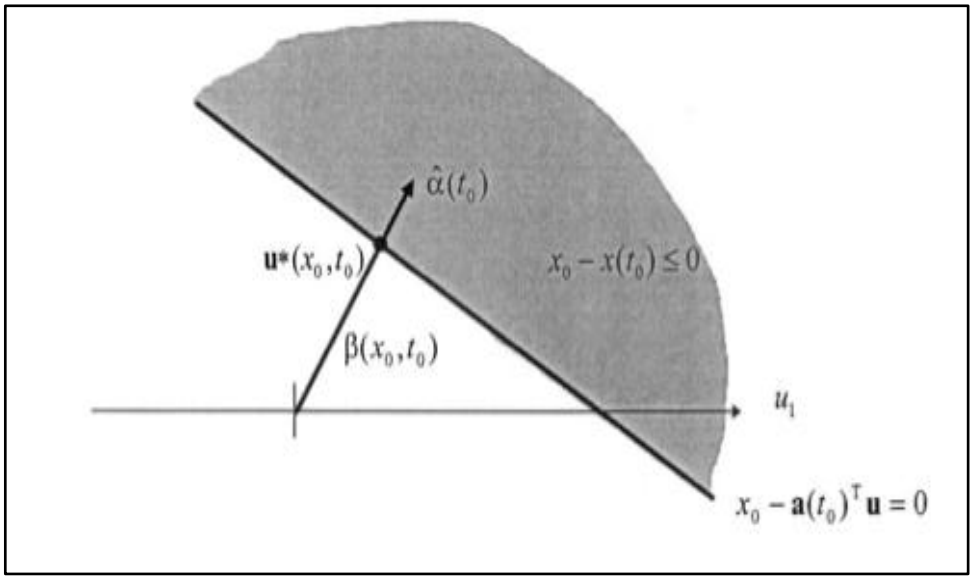

Figure 17 Geometric representation of time t_0

According to the well-established terminology of the theory of structural reliability briefly describe in pervious chapter, the limit state function is

$$g_0\,(\mathbf{u}) = x_0 - A(t_0)^T U \tag{5.23}$$

and the limit-state surface is g_0 **(u)** = $x_0 - A(t_0)^T U = 0$, $\widehat{\propto (t_0)}$ is the unit outbound normal vector (towards the failure set), and $\beta(x_0, t_0)$ is the reliability index for the event $\{X(t_0) \geq x_0\}$.

Among all realizations of **U** that give rise to the event $\{X(t_0) \geq x_0\}$, the one that has the highest likelihood is the one nearest to the origin (see Fig. 5.5). This point, known as the "design point" in the theory of structural reliability, is given by

$$U^*(x_0, t_0) = \beta(x_0, t_0).\widehat{\propto (t_0)}$$

$$U^*(x_0, t_0) = x_0 \frac{a(t_0)}{\|a(t_0)\|^2} \tag{5.24}$$

The corresponding "design point" excitation $F^*(t)$ and response $X^*(t)$ are

$$F^*(t) = S(t)^T U^*(x_0, t_0)$$

$$F^*(t) = x_0 \frac{S(t)^T a(t_0)}{\|a(t_0)\|^2} \tag{5.25}$$

and

$$X^*(t) = a(t)^T U^*(x_0, t_0)$$

$$X^*(t) = x_0 \frac{a(t)^T a(t_0)}{\|a(t_0)\|^2} \tag{5.26}$$

Note that $X^*(t) = x_0$ For the linear system under Gaussian excitation, the design point realization is proportional to the threshold x_0. Of most interest, is the design point excitation $F^*(t)$, which is that specific realization of $F(t)$ hat has the highest likelihood to give rise to the event $\{X(t_0) \geq x_0\}$. This realization is of particular interest from the viewpoint of design, as one can assure safety by providing adequate capacity against this particular deterministic excitation. Typically, $F^*(t)$ is a nearly harmonic, gradually intensifying time function with a frequency close to the fundamental frequency of the system. After solving the optimization problem, the non- Gaussian response is replaced by a Gaussian one which is defined by the based function vector $a(t_x) = \nabla_u X(t_x)|_{u=u^*(x_0,t_x)}$ and the probability of failure could be

expressed by $\phi(-\beta(x_0, t_0))$, where β is called reliability index and is equal to Euclidean norm of U^*, where ϕ(-) is the standard normal CDF, and for obtaining $a(t_0)$, simple manipulation of (5.24) yields:

$$a(t_0) = x_0 \frac{U^*(x_0,t_0)}{\|U^*(x_0,t_0)\|^2} \tag{5.27}$$

It is evident that knowing the projection point $U^*(x_0, t_0)$, one can determine the gradient vector $a(t_0)$. Note that $a(t_0)$ does not depend on the threshold x, as is evident in (5.21), even though the latter appears on the right-hand side of (5.27). This is because $U^*(x_0, t_0)$ is proportional to x for a linear system. The obtained vector from the above equation separated to m vectors a_1 to a_m each with n elements.

Then the IRFs of TELS can be obtained from the following equations [1]

$$\sum_{k=1}^{n} h_j(t_n - t_k) S_j^i(t_k) \Delta t \cong a_j^i(t_n) \tag{5.28}$$

j = 1,2,….m

i = 1,2,…n

Each of the above relations represents a set of n equations that can be solved for the values of the IRFs at time points. The obtained IRFs indicate TELS for the specified threshold X and time point t_n and define a linear system in the space of u variables that has an identical design point with the nonlinear system. By obtaining the IRFs or frequency response functions (FRFs) (by the Fourier transform of IRFs) of equivalent linear system, linear random vibration methods can be used to determine the considered statistical responses for the nonlinear system.

Note: Having determined the IRF of the linear system for the particular input–output pair, we can determine the corresponding frequency response function (FRF) by the Fourier transform [2]

$$H(\omega) = \int_0^\infty e^{-i\omega t}\, dt \tag{5.29}$$

5.4 Define the Tail Equivalent linear System (TELS)

Consider a multi-degree-of-freedom (MDOF) nonlinear system defined by the 2nd order differential equation

$$M\ddot{Y} + C\dot{Y} + R(\dot{Y}, Y) = PF(t) \tag{5.30}$$

where Y denotes the vector of displacements, M denotes the mass matrix, C denotes the viscous damping matrix, R denotes the restoring force vector, and P denotes a load distribution vector. The system is subjected to the stochastic excitation F(t), which is discretized in the form of (5.5), [1] with initial conditions Y(0) = 0 and $\dot{Y}(0) = 0$. Although TELM can be developed for a system subjected to multiple excitations, bt in our study we are doing for single excitation process. The restoring force vector is a function of the displacement vectors and velocity vectors, allowing the system to have a hysteretic behavior. Suppose we are interested in a generic response X(t) of the system, which most generally is defined as a nonlinear function of the nodal displacement and velocity vectors, i.e., $X = X(Y,\dot{Y})$. As a special case, X(t) could be a nodal displacement, in which case it is equal to the corresponding element of Y. Restoring force is function of $\dot{Y}$ & Y for allowing the system to have a hysteretic behavior. With the discretize form of the excitation as described earlier. The vector Y &$\dot{Y}$ are implicit function of the random variable u.

Now we want to determine the tail probability $P_r(x \leq X(t_n, u)$ for a special threashold x and time tn. Using the well defined method of structural reliability theory, described in the previous section, tail probability section are determined, and using that determine the design point of given non linear system at x_1 threshold and t_1 time is $u^*_{NL}(x_1, t_1)$ after determine the design point of non linear system we use the main principal of tail equivalent linearization method that is tail probability of the linear response to a first order approximation of tail probability of the non linear response. It implies that design point of nonlinear system is similar to design point of the linear system, design response of the non-linear system is similar to design point of linear system and also design excitation of nonlinear system is similar to design point of linear system. We are

interested in equivalent linear system, for getting this according to main principal of TELM we are getting equivalent linear system by replacing the nonlinear system excitation by design point excitation and also make nonlinear parameter is linear so that equation (5.30) is replace by equation (5.31)

$$M\ddot{Y} + C\dot{Y} + K(Y) = PF^{*}(t) \tag{5.31}$$

Equation (5.31) is an equation of equivalent linear system corresponding to given nonlinear system equation (5.30). Equation (5.31) is nothing but an equation of Tail Equivalent Linear System [1].

Suppose take a non-linear system and compute the design point of this system $u_{NL}^{*}(x_1, t_1)$. Then make a list of linear system possible in u-space and compute a design point of all linear system in list. In this list a system which have a design point similar to the design point have given nonlinear system. System in the listed, which have design point is similar to design point of nonlinear system is called tail equivalent linear system, but it's not a practically easy job to make a list of the possible linear system in given u-space. Instead of making list we make given nonlinear system as a linear system by replacing the excitation with design excitation according to the design point of non-linear system, we now proceed with the formal definition of the tail equivalent linear system (TELS). For the response $X(t, u)$ of a nonlinear system to stochastic excitation $F(t)$ represented in the discretized form [1], the TELS for specified threshold x and time t_n is a linear system that, in the space of the standard normal random variables u, has the same design point as the nonlinear system. This equivalence implies that the tangent plane of the nonlinear system response at the design point coincides with the hyper plane of the linear system response for the same threshold and time. It follows that the FORM approximation of the tail probability of the nonlinear system response is identical to the tail probability of the response of the TELS. This property encourages the term Tail-Equivalent Linearization Method (TELM).

The above definition leaves out specifics as to what the form of the linear system is, how many degrees of freedom it has, what its parameters are, and which of its responses is to be considered. However, as we will shortly see, these specifics need not be considered.

Now we explore various other options for selecting the equivalent linear system.

As is commonly done in the conventional ELM, consider the equivalent linear system as a 2nd order system governed by the differential equation

$$M\ddot{Y} + C\dot{Y} + K_e(\theta)Y = PF(t) \tag{5.32}$$

where $K_e(\theta)$ is an equivalent stiffness matrix defined in terms of a set of parameter θ. According to the above definition of the TELS, the parameters θneed to be selected such that $u^*_{LN}(x_1, t_1) = u^*(x, t_n)$, where $u^*_{LN}(x_1, t_1)$is the design point of the linear system. However, there is no guarantee that, for any values of θ, the design point of the linear system will coincide with the design point of the nonlinear system that is $u^*_{NL}(x_1, t_1) \neq u^*_{LN}(x_1, t_1)$ unless the size of θ matches the size of u and a valid solution for the set of nonlinear equation is found. One way to overcome the difficulty is to use an approximate tail equivalent linear system by minimizing norm of the error in matching two design points.

To refine the approximation one may increase the number parameter θ by for example replacing the mass and damping matrices with parameterized ones, or by considering a higher-order (3rd order, 4th order, etc.) linear system. On the other hand, to simplify the solution, one may consider a simpler equivalent linear system, e.g., a single-degree-of freedom (SDOF) oscillator with equivalent mass, damping and stiffness characteristics, even when the nonlinear system has multiple degrees of freedom. Naturally, the error in matching the design points will depend on the number of parameters defining the equivalent linear system. The approach described in the preceding paragraph can be considered a parametric linearization method. The real strength of the TELM lies in the fact that we need not consider any of the parameterized systems described, nor do we need to solve the optimization problem. Observe that the design point of the nonlinear system is the point of projection from the origin onto the tangent plane. As was demonstrated in the preceding section, knowledge of this point on a hyper plane completely defines the IRF of the corresponding linear system. Specifically, knowing the design point $u^*_{LN}(x_1, t_1) = u^*(x, t_n)$ we can use (5.27) and (5.28) to determine the IRF of the TELS for the particular response threshold x and time t_n. We cannot tell what the order of the linear system is, or what its parameters are. But these information are not necessary since the IRF completely characterizes the linear system for the particular excitation and response pair. In contrast to the conventional ELM and the method

described in the preceding paragraph, the approach described in this paragraph may be regarded as a nonparametric linearization method.

A question that may arise is whether any hyper plane in the space of u defines a linear system that is causal and stable. As mentioned earlier, for a causal system one must have h(t) = 0 for t < 0. The set of equations in (5.28) do not involve h(t) for t < 0 and, therefore, we can impose this requirement on h(t) obtained for any hyper plane in the u space. However, an arbitrary hyper plane in the u space may not satisfy the stability requirement, $\int_0^\infty h(t)dt < \infty$.

For example, when the excitation is a discretized white-noise, a hyper plane with its unit normal having equal components yields a function h(t) that remains constant in time regardless of the number of time steps n. Such a system obviously is not stable. To investigate the stability condition for the TELS obtained from the design point of a nonlinear system, let $f_i^* = F^*(t_i)\Delta t$ denote the discretized design-point excitation at time step t_i . It is shown in preceding section that for the IRF determined from the tangent plane of the nonlinear system at $u^*, h(t_n - t_i) \propto \partial X(t_n)/\partial f_i^*$

That is, the IRF of the TELS at time $t_n - t_i$ is proportional to the sensitivity of the design-point response of the nonlinear system at time t_n with respect to the discretized excitation value at time t_i . Provided

$$\lim_{n\to\infty} \sum_{i=0}^{n} \partial X(t_n)/\partial f_i^* \Delta t$$

The resulting TELS will be stable. Of course, it would be difficult to verify this condition for a general nonlinear system. However, for many structural systems of interest, one would expect the response sensitivity with respect to the earlier load value to be finite and diminish to zero with increasing $t_n - t_i$. For a given nonlinear system, this condition can be checked numerically by deterministic dynamic analysis at the designpoint excitation. This was done for the example non-degrading hysteretic systems considered in preceding section.

Chapter 6 Random vibration analysis on TELS

6.1 Introduction

By repetitive TELM analysis, a series of design points for an ordered set of threshold $x_1<x_2<x_3..... <x_p$ at a specific time t_x is obtained. Once TELS obtained linear random vibration analysis of the TELS, any one or both in time or frequency domain, is implemented to determine the statistics of interest for the specified threshold of the nonlinear response in first-order approximation. These statistics include the tail probability at a given time, the mean rate of up-crossing the threshold, and the tail probability of the extreme response over the duration of the excitation (the fragility). From the time when the TELS is a linear system and the excitation is zero-mean Gaussian, the response of the TELS is also zero-mean Gaussian and the existing results for such a process can be used. Though, as the threshold changes, the TELS also changes so that the approximate distribution obtained for the nonlinear response is non-Gaussian. In essence, the statistics for different thresholds are approximated by different Gaussian distributions.

6.2 Analysis Methods

In this section we describe analysis methods for both stationary and non-stationary responses. Although earthquake ground motions are inherently non-stationary, results based on stationary analysis are also presented because the stationary response over a short time period can provide a useful and simple approximation.

6.2.1 Stationary response

Stationary random vibration analysis for a given threshold x is carried out most conveniently in the frequency domain by use of the FRF of the TELS, $H(\omega, x)$.

As is well known, most statistics of interest for a zero-mean, stationary Gaussian response process are given in terms of the low-order spectral moments [7]

$$\lambda_m(x) = \int_0^\infty \omega^m |H(\omega, x)|^2 S_{AA}(\omega) d\omega, \qquad m = 0,1,2 \qquad (6.1)$$

where $S_{AA}(\omega)$ denotes the one-sided power-spectral density of the ground acceleration. The FRF $H(\omega,x)$ used in (6.1) can be that of the TELS for the nonlinear response to the exact stationary excitation, or that obtained for a white-noise, if the true excitation is broad-band. Note that the computed spectral moments are functions of the specified response threshold due to the dependence of the FRF on x.

Since the response of the TELS is zero-mean Gaussian, the first-order approximation of the tail probability of the nonlinear response at threshold x is given by

$$\Pr[x<X(t)] \cong 1 - \Phi\left[\frac{x}{\sqrt{\lambda_0(x)}}\right] \qquad (6.2)$$

Note that due to the dependence of $\lambda_0(x)$ on x, the distribution of the nonlinear response is not Gaussian.

The mean rate of up-crossing level x is determined by use of the well-known formula for a zero-mean Gaussian process

$$\upsilon^+(x) \cong \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_0(x)}}\, exp\left(-\frac{x^2}{2\lambda_0(x)}\right) \qquad (6.3)$$

Again, because of the dependence of the spectral moments on x, the above result is different from the up-crossing rate of a Gaussian process.

The tail probability of the maximum absolute response over a time interval τ, denoted by $Pr\left[x < max_{(t,t+\tau)}\|X(t,u)\|\right]$ may be found by any of the approximate solutions available for a zero-mean stationary Gaussian process [16]. A simple and accurate approximation due to Vanmarcke is

$$Pr\left[x < max_{(t,t+\tau)}\|X(t,u)\|\right] \cong$$
$$1 - \left\{1 - exp\left[-\frac{r^2(x)}{2}\right]\right\} \times exp\left[-\frac{\tau}{\pi}\sqrt{\frac{\lambda_2(x)}{\lambda_0(x)}}\,\frac{1-exp\left[-\sqrt{\pi/2}\,\delta(x)^{1.2}r(x)\right]}{exp\left[r^2(x)/2\right]-1}\right] \qquad (6.4)$$

where $r(x) = x/\sqrt{\lambda(x)}$ and $\delta(x) = [1 - \lambda_1^2(x)/\lambda_0(x)/\lambda_2(x)]^{1/2}$.

Note again that the resulting distribution is different from Vanmarcke's distribution for the extreme of a zero-mean stationary Gaussian process because the spectral moments here are functions of the threshold *x*.

Now suppose that the ground acceleration is scaled by a factor *c* so that its power-spectral density is $c^2 S_{AA}(\omega)$. It is clear from (6.1) that all the spectral moments will be scaled by the factor c^2. Hence, the response statistics for the scaled problem are obtained from (11)–(13) by simply replacing $\omega_m(x)$ in these expressions by $c^2\,\omega_m(x)$. In particular, the fragility curve for a given response threshold *x* is obtained by replacing $r(x)$ in (6.4) by $r(x)/c$ and plotting the resulting expression as a function of *c*.

6.2.2 Non-stationary response

For non-stationary response, time-domain analysis with the IRF of the TELS can be performed to compute the statistics of interest. Such analysis in general requires two-dimensional integrations involving the IRF and the autocorrelation function of the excitation. An alternative, more convenient approach, using reliability methods is described below.

Let $\mathbf{u}^*(x, t_n)$ represent the design point for the response threshold *x* at time t_n. If the TELS is obtained using the actual input non-stationary excitation, then this design point is already available.

If, on the other hand, the TELS is that of a substitute excitation, such as white-noise, then the design point for the actual excitation must be obtained. This is done by first computing the elements of the n-vector $\mathbf{a}_n(x) = [a_1(t_n,x)...a_n(t_n,x)]$,

$$a_i(t_n, x) = \int_0^{t_n} h(t_n - \tau, x) s_i(\tau) d\tau \tag{6.7}$$

where $h(t,x)$ is the IRF of the TELS obtained for the surrogate excitation (white noise), and then computing an approximation of the design point for the true input excitation from

$$u^*(x, t_n) \cong \frac{x}{\|a_n(x)\|} \frac{a_n(x)}{\|a_n(x)\|} \tag{6.8}$$

With the design point available, the statistics of the non-stationary response are computed as follows:

A first-order approximation of the tail probability at time t_n is given by

$$\nu^+(x, t_n) = \lim_{\delta t \to 0} \frac{Pr[-g(u, x, t_n) \leq 0 \cap g(u, x, t_n + \delta t) \leq 0]}{\delta t}$$

with a finite but small δt. The numerator in the above expression represents a parallel system reliability problem with two components. This probability is easily computed by FORM by translating the known design point excitation at time t_n to obtain the design point excitation at time $t_n + \delta t$.

Finally, the tail probability of the maximum absolute response over duration τ is obtained by reformulating the problem as a series-system reliability problem:

$$Prx < \max_{t,t+\tau} X(t, u) \geq Pr\{\cup_{t_i \epsilon (t,t+\tau)} [x < X(t_i, u)]\} \tag{6.9}$$

The first line in the above expression describes the tail probability of the extreme response as the probability that the response will exceed the specified threshold in any of the time steps within the interval (t, t+δt). This obviously provides a lower bound; however, for a small step size, the series system probability provides a fairly accurate representation. The second line represents the FORM solution of the series system

problem in terms of the multinomial cumulative distribution function (CDF), where k denotes the number of time steps within the interval (t, t+ δ) $\boldsymbol{B}(x) = [\beta(x,t1),\ldots,\beta(x,tk)]$ is the set of reliability indices, and R(x) is a k×k correlation matrix having the elements $\rho_{ij}(x) = \alpha(x,t_i)\cdot\alpha(x,t_j)$, i, j=1,...,k, where $\alpha(x,t_i)$ is the unit normal vector at the ith time step. This solution clearly requires finding the design points for all the time steps within the interval.

Chapter 7 Example Application of TELM on nonlinear system

Numerical example is considered in this section. Example deals with the response of the linear oscillator to Gaussian and non- Gaussian excitation comparisons are made with Gaussian process as non- Gaussian process. This section is also shows numerical investigation of TELS.

Example-7.1 Consider the response of a linear oscillator described by the differential equation

$$\ddot{x} + 2\xi_n\omega_n\dot{x} + \omega_n^2 x = f(t) \tag{7.1}$$

Where ω_n is the natural frequency and ξ_n the natural damping ratio of the oscillator. The unit-impulse-response function of the system is given below

$$h_f(t) = \exp(-\xi_n\omega_n t)\left[\left(\frac{(2{\xi_n}^2-1)\omega_f}{\sqrt{1-{\xi_n}^2}}\ \sin\left(\omega_n\sqrt{1-{\xi_n}^2}t\right)\right) - 2\xi_n\omega_n\cos\left(\omega_n\sqrt{1-{\xi_n}^2}t\right)\right] \tag{7.2}$$

In the following analysis $\omega_n = 4\pi$ rad/sec & $\xi_n = 0.05$ are used

Let y(t) be a Gaussian process with zero mean and unit variance. We consider three excitation process f(t) defined by

Process 1 : f(t)=y(t)

Process 2 : f(t)= y(t)$|y(t)|^{\theta}/\sqrt{E[|y(t)|^{2+2\theta}]}$

Process 3 : f(t)= $\exp[\lambda + \xi y(t)] - m$;

where m>0, $\xi = \sqrt{\ln(1 + \frac{1}{m^2})}$ and $\lambda = \ln m - 0.5\xi^2$

In figure 18 shows the excitation is a Gaussian white noise sample realization and in figure 19 give the sample realization of unit impulse response function of the system.

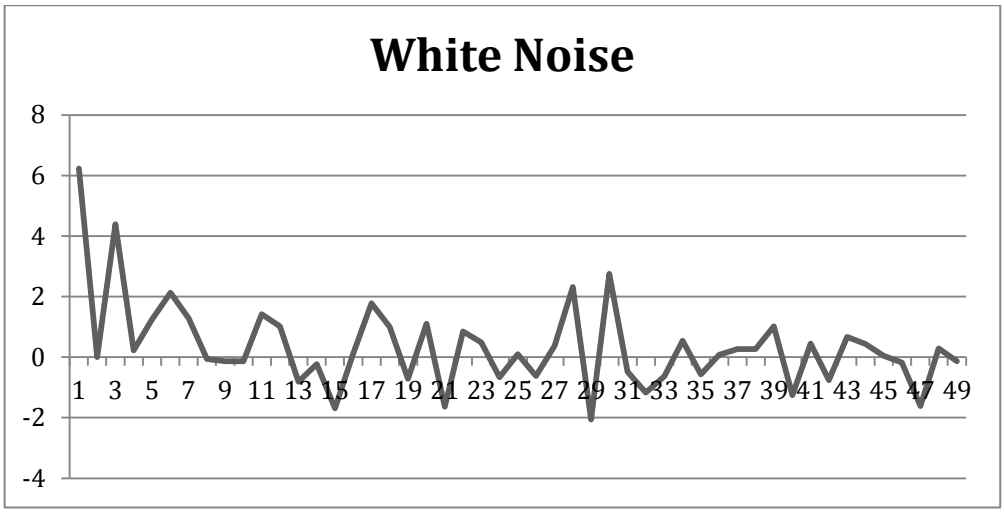

Figure 18 Realization of white noise

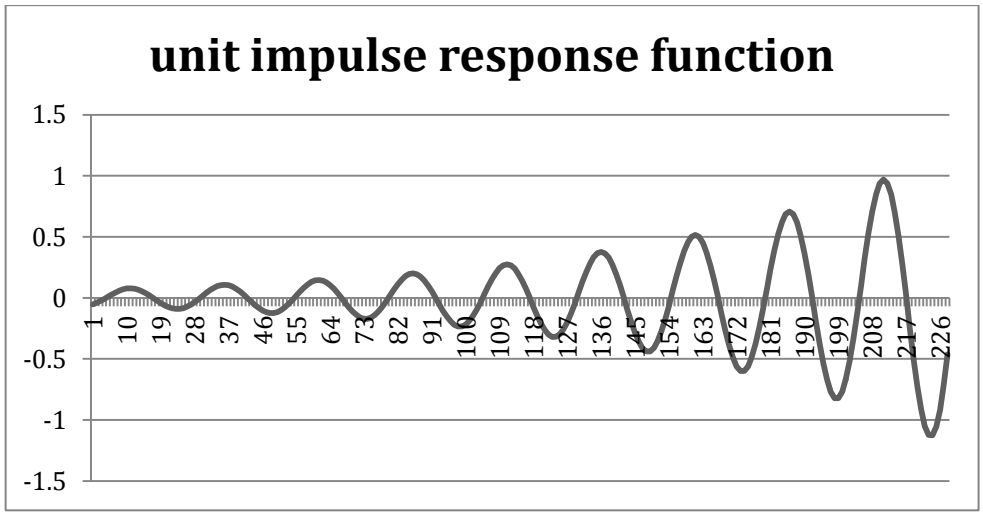

Figure 19 impulse response function of the filter

All processes have zero mean and unit variance. Process 1 represents a Gaussian excitation, while processes 2 and 3 represent non-Gaussian excitations obtained by transformations of y(t). One can easily verify that the latter two processes approach the Gaussian process as $\theta = 0$ and m=∞. Hence, the parameters θ and 1/m describe the degree of non-Gaussianity of f(t). It is noted that process 3 is a shifted lognormal process.

The first-order probability density functions of the three processes for θ =1 and m=1 are shown in Fig. 7.3(a,b&c).

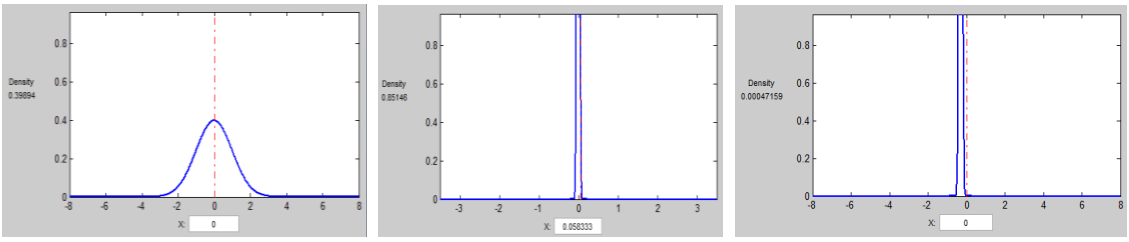

Figure 7.3(a,b,c) probability density function for process 1,2 and 3 respectively.

It shows that process 2 has a symmetric distribution with infinite density at the zero point, whereas process 3 has an asymmetric distribution and is bounded from below,and the cumulative density functions of the three processes for θ =1 and m=1 are shown in Fig. 20.

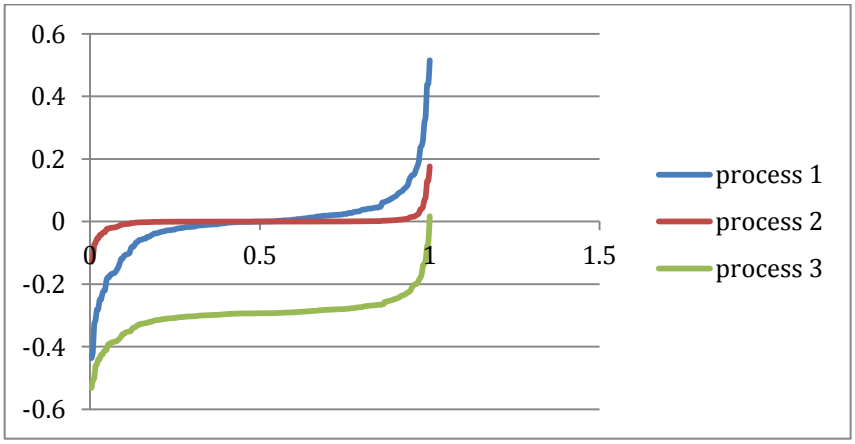

Figure 20 cumulative density function of all three process

The Gaussian process $y(t)$ is represented in the discretized form of Eq. (1) by filtering a train of random pulses of magnitude u_i, i=1,2,…, with normalized basis functions

$S_i(t) = \sigma \int_{t_{i-1}}^{t} h_f(t-\tau)\ d\tau\ \ t_{i-1} < t \leq t_n$

where t_i are equally spaced time points with t_1=0 and Sample realizations of the three processes for θ =1 and m= 1 are shown in Fig. 20(a,b,&c).

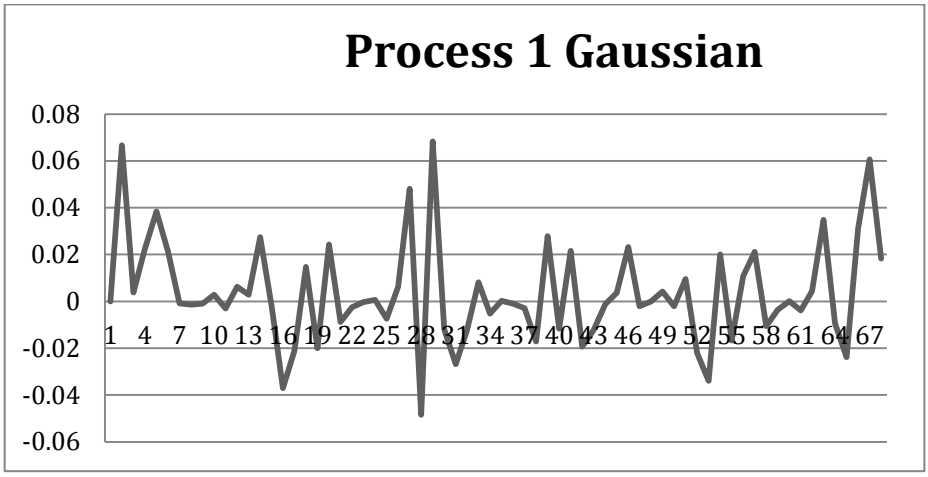

FIG 7.4(A)

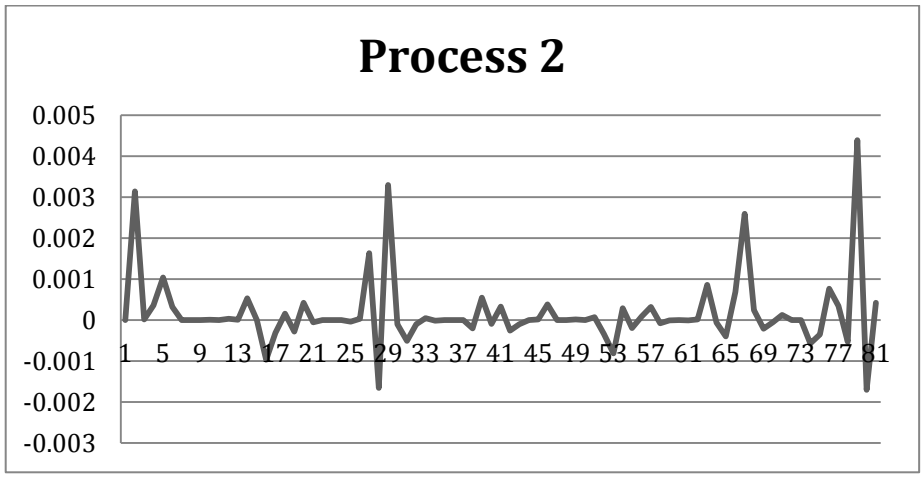

Fig-7.5(b)

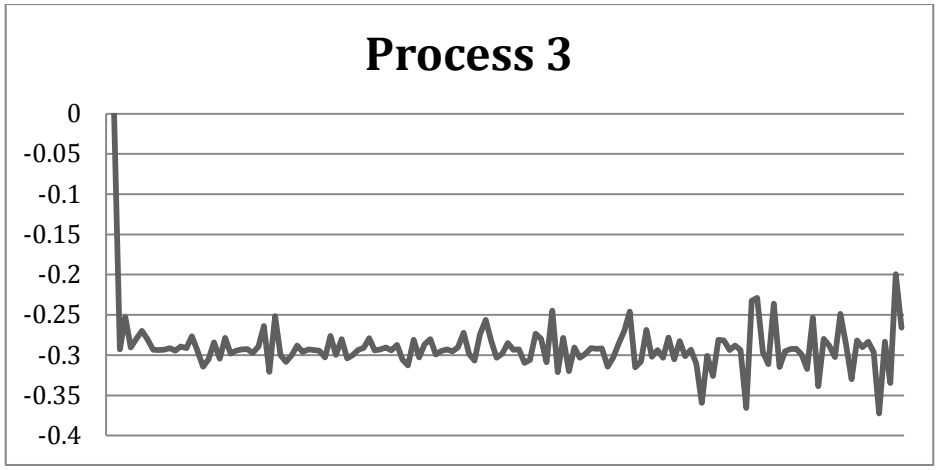

Fig-7.5(c)

Figure 21(a,b&c) sample realization of three process with function of θ =1 and *m*= 1

Fig 22 shows the generalized reliability index $\beta = \phi^{-1}\{P[x(t_0) > x_0\}$ for t_0=5 sec and x_0= 0.1 as the function of θ and 1/m as computed by FORM. its shows at $\theta = 0$ and 1/m =0 the excitation process 2 and 3 are Gaussian. And the result of the all three case coincides. With increasing θ and 1/m the excitation process 2 & 3 become increasing non-Gaussian. This is marked by increasing departure of FORM result from the Gaussian case (process 1) it is evident that the limit state surface for the response to excitation process 2 is more strongly non-linear as the FORM results for this case.

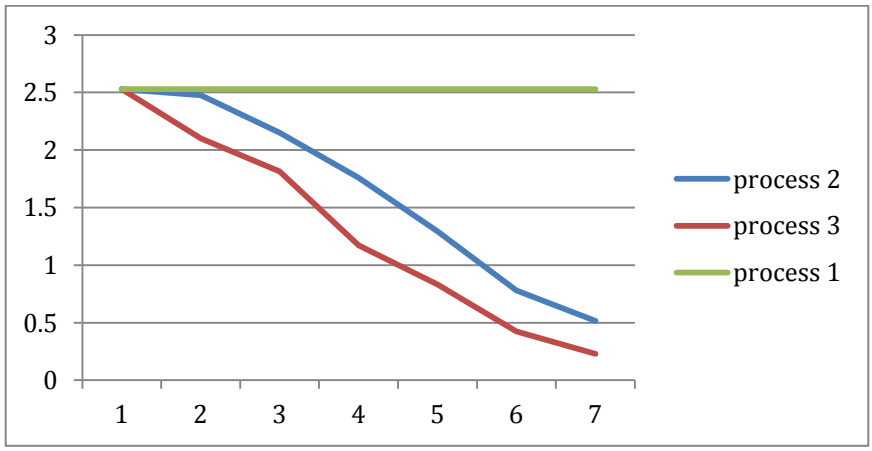

Figure 22 Generalized reliability index f or {{x(5) >0.1} as a variable of the degree of non-Gaussianity of the excitation

Figure 23 shows that the general reliability index for t_0=5 sec as a function of x_0 for $\theta = 1$ and m=1 as computed by FORM. As expected for the response of Gaussian excitation reliability index is proportional to threshold value. For excitation 2 the response distribution is symmetric and from the past study we know the response distribution from 2nd order reliability method and Monte Carlo simulation are shown only for positive values of threshold. It is evident that FORM is resulted more accurate for higher threshold. This is due to the asymptotic nature of these approximations. The poor performance of FORM for excitation process 2 is due to the infinity density at the origin which strongly wraps the limit state surface particularly for small thresholds.

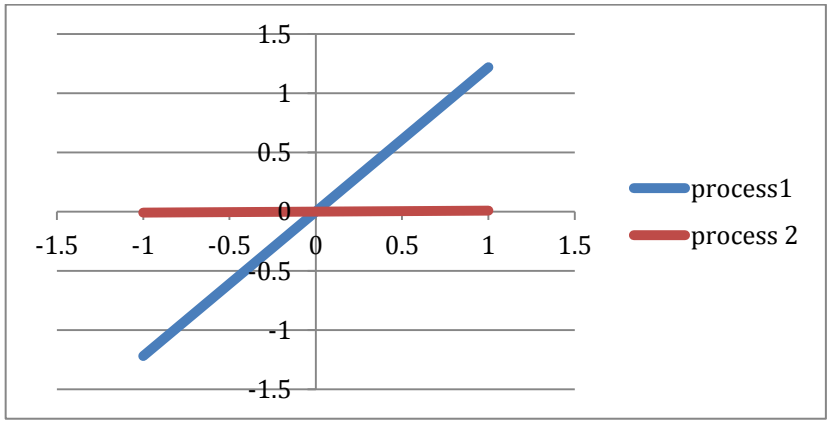

Figure 23 generalized reliability index for event $\{x(5)>x_0\}$ for $\theta = 1$ and m=1 as a variable x_0

Fig 24 and fig 25 shows that the design point excitation and the design point response for the event $\{x(5)>x_0\}$, respectively.

It is interesting the most likely realization of excitation corresponding to process 2 as a zero value for the first few seconds and the most likely realization of excitation corresponding to process 3 as a constant negative value for first few seconds. All three-design point excitations are oscillatory in nature and gradually build up to a large peak just before the time t_0.

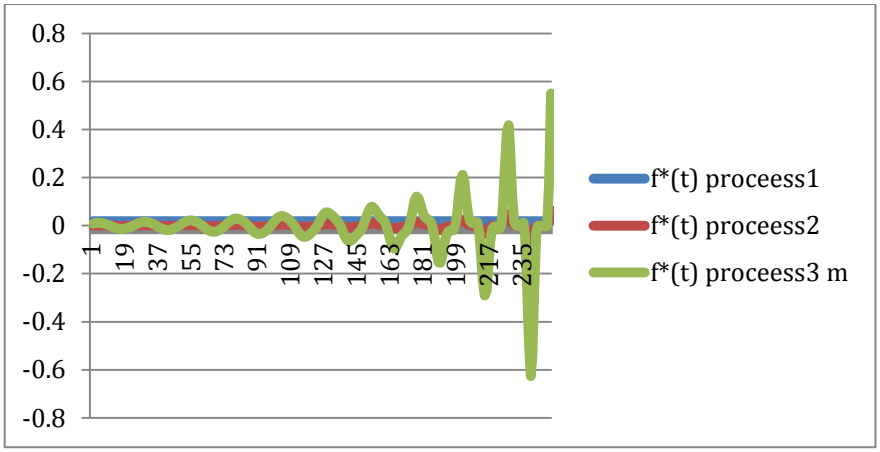

Figure 24 design design point for the event {x(5)>x_0}

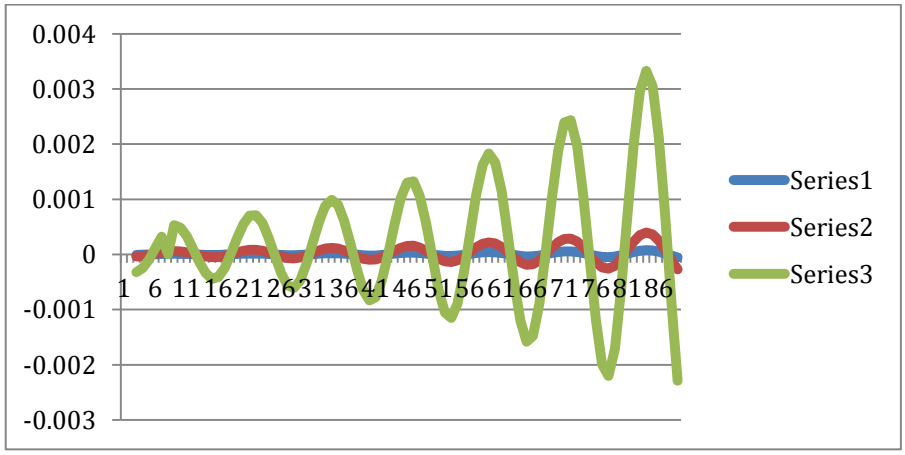

Figure 25 the design point response for the event {x(5)>x_0}

Fig25 response corresponding to design point excitation

fig -26(a, b &c) shows the probability density functions of the response $x(t_0)$ at t_0=5 s obtained by FORM p with respect to x_0. Note that in the semi-logarithmic scale shown, the normal density appears as a parabola. The distinct non-Gaussian nature of the response to excitation processes 2 and 3 is clearly evident.

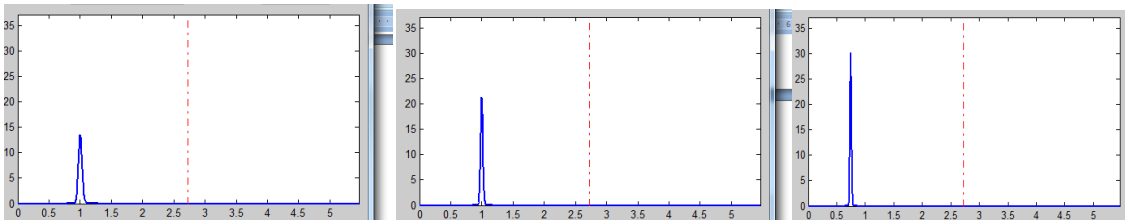

Figure 26 probability density function of response x0(t0) at t0 = 5 sec. for for θ=1 and m=1 as a variable

Example 7.2 We consider the corner of a six-storey building subjected to earthquake-induced ground acceleration. Time response vector of interest is $x(t)=[m_1(t)\ m_2(t)\ p(t)]^T$ where $m_1(t)$ and $m_2(t)$ denotes the bending moment around axial force in the column and p(t) denotes the axial force in the column. The failure domain for the column is

$$G(x) = 1.33 - \sqrt{\left(\frac{m_1}{m_0}\right)^2 + \left(\frac{m_2}{m_0}\right)^2} - 0.33\left(1 - 3\left(\frac{p}{p_0}\right)\right)^2 \qquad (7.1)$$

Where m_0 p_0 are capacity parameters. The input ground acceleration is applied along axis 2. The structure is assumed to be linear and each member of the response is obtained by modal superposition in the form $x_i(t)=\mu_i + \sum_{j=1}^{k} b_j q_j(t)$.

Where μ_i is the static response due to dead load, k is the number of modes, b_j are effective modal participation factor and $q_i(t)$ are normalized modal responses for the corresponding modal frequencies and damping ratio of the structure. The factor b_j for three responses, as normalized by the parameter m_o or p_0 and the frequencies and damping ratio for the first 4 modes of the structure are listed in table -1

TABLE 7.1 MODAL PROPERTIES OF EXAMPLE STRUCTURE

Mode	**b_j/m_o for m_1 (t)**	**b_j/m_o for m_2 (t)**	**b_j/m_o for p(t)**	**ω_j (rad/sec)**	**Damping**
1	-2.90	4.30	3.02	8.74	0.05
2	-2.83	-3.26	-2.41	8.95	0.05
3	-0.0754	-1.04	0.124	11.1	0.05
4	-3.21	4.65	-1.29	26.8	0.05
5	3.14	-3.55	1.30	27.5	0.05

The limit surface defined by eq (5.1) is parabolic in the space of p and m_1 and m_2 and it is circular in the space of m_1 and m_2 or p. the fact that it is a closed surface calls for caution in the use of FORM approximations.

We first examine the probability of the event {G(x(t_0)<0} at t_0 =5 sec. the design point for this event are

$m_1^* = 0.78\ m_0$,

$m_2^* = 0.88\ m_0$

$p^* = 0.46\ p_0$.

These value represent the most likely set of realization of $m_1(t)$, $m_2(t)$ and p(t) to cause failure of the column at t = t_0 = 5 sec.

Estimate of the generalized reliability index based on FORM is 2.54. The small difference between these estimates is an indication that in the space of the standard normal variable u. the limit state surface is nearly flat in the neighborhood of the design point. Further, while the surface remains closed in the u space, only the region close to the design point makes a dominant contribution.

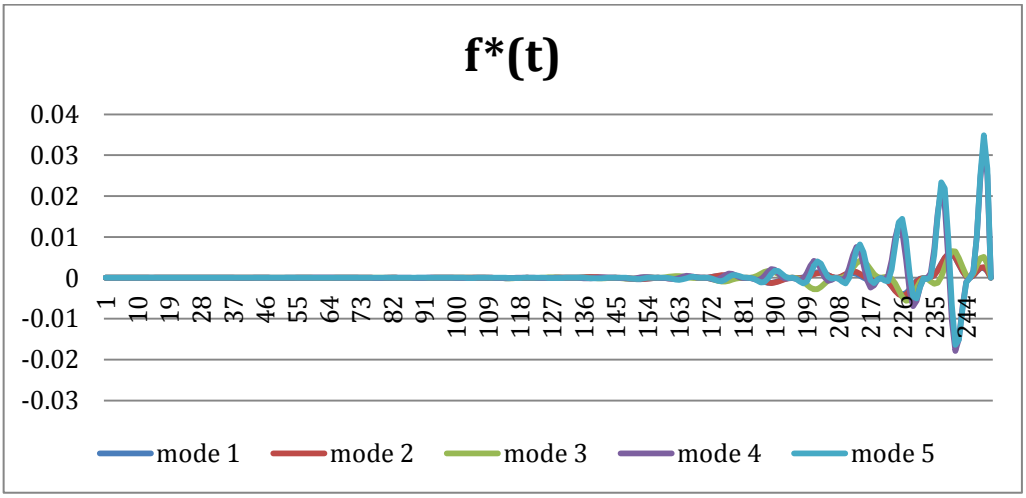

Figure 26 design point excitation for different modes

Shown in fig (26) design point excitation of different modes of six storey column. We observed in this when modal participation factor is increased design point excitation is also increased its corresponding to calculation in given APPENDIX –I.

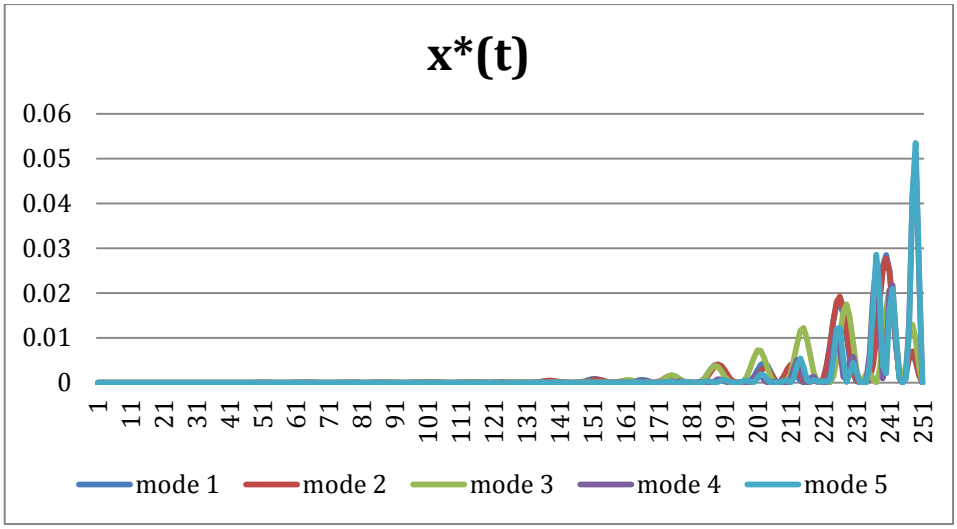

Figure 27 Design point response corresponding to different modes

Fig-27 shows the response corresponds to design point with different modes. It shows by increasing the fundamental frequency of system. Its value is also increased.

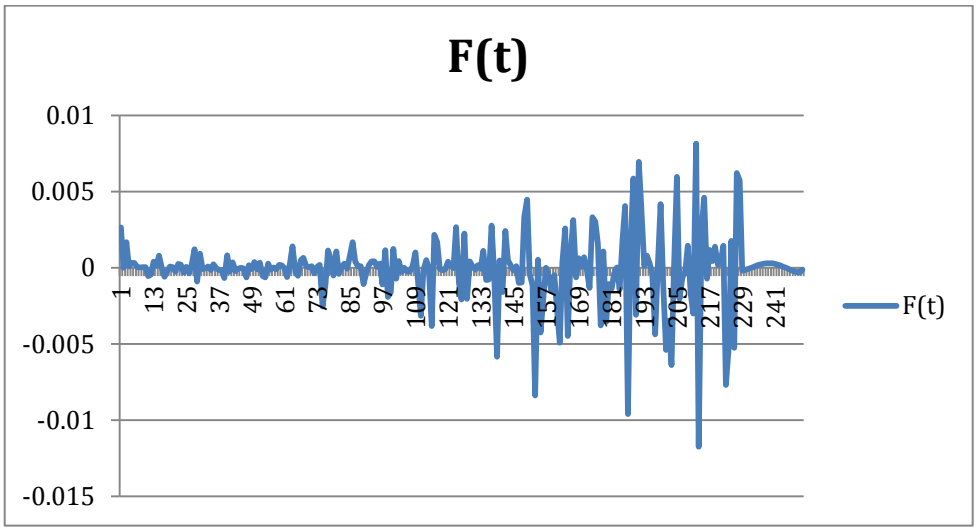

Figure 28 simple realization of the process f(t)

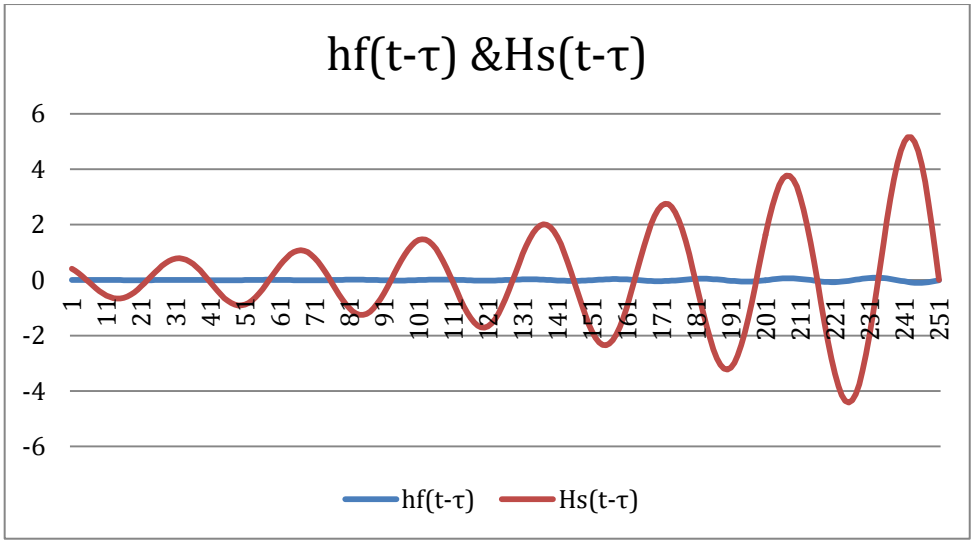

Figure 29 impulse response function of filter and system.

In fig-29 represents the impulse response function of ground and system. Calculation are given in APPENDIX -I

Chapter 8 Summary and Conclusion

A new linearization method for random vibration analysis is described. It is based on the first order reliability method. the equivalent linear system is defined by matching its designs point and the standard of normal variable with that of non linear system for a specified threshold and time.

The following steps summaries the key point of the TELM.

- Discretize the input excitation with the help of the concept of time modulating function.
- Use concept of FORM equivalent linear system is defined by matching its design point is the space of standard normal random variable with that of nonlinear system for specified threshold and time.
- Shows that the knowledge of the design point uniquely determine the equivalent linear system in terms of its impulse response function, this determination is in a numerical non-parametrical form. There is no need to characterize the linear system in terms of its order degree of freedom or parameters.
- Numerical investigation shows that TELS depends on threshold, excitation and time.
- TELM can be applied to non stationary excitations by defining a time varying TELS.
- Once the TELS for a sequence of threshold are determine, method of linear random vibration analysis are used to compute various statistics of response such as CDF,PDF at a given time, mean crossing rate and the distribution of the maximum response over an interval.
- TELM required considerable more analysis than ELM, if one is only interested in second moment of the response.
- If one is only interested in second moment analysis, ELM is the appropriate method to use, where as TELM is the appropriate method for computing the distribution and statistics of the extreme response.
- TELM is not affected by the complexity of the nonlinear system.

APPENDIX 1

Calculation done on MS excel for example given in chapter 7.

Example 1

									0.140692315
							a(t)=	si(t)*h(t-τ)	
							si(t)=	σ*hf(t-τ)	0.003281958
							σ=	(2*π*S*Δt)^0.5	0.023327201
				3.23E-05					
			hf(t-τ)						
	τ	part 1	part 2	part 3	part 2+3	(part 2+3)	s(t)	normalized s(t)	a(t)
	0	0.043218	0.031726664	-1.255363	-1.22364	-0.05288			
	0.02	0.043764	0.278227231	-1.20602	-0.92779	-0.0406	0.012279	0.004351577	-0.000176693
	0.04	0.044318	0.507263385	-1.080975	-0.57371	-0.02543	0.015179	0.005379328	-0.000136773
	0.06	0.044878	0.704458453	-0.888077	-0.18362	-0.00824	0.017185	0.006090444	-5.01883E-05
	0.08	0.045446	0.857434438	-0.639434	0.218	0.009907	0.018148	0.006431551	6.37185E-05
	0.1	0.04602	0.956588988	-0.350654	0.605935	0.027885	0.017978	0.006371495	0.000177672
	0.12	0.046602	0.99569814	-0.039863	0.955835	0.044544	0.016659	0.005903893	0.000262984
	0.14	0.047192	0.972306999	0.27343	1.245737	0.058788	0.014244	0.005048172	0.000296774
	0.16	0.047788	0.887883837	0.56956	1.457444	0.069649	0.010861	0.003848982	0.000268078
	0.18	0.048393	0.74772792	0.829939	1.577667	0.076348	0.006699	0.002374	0.000181249
	0.2	0.049005	0.560636882	1.038222	1.598859	0.078352	0.002004	0.000710198	5.56451E-05
	0.22	0.049624	0.338354487	1.181335	1.51969	0.075414	-0.00294	-0.001041205	-7.8521E-05
	0.24	0.050252	0.094833472	1.250296	1.34513	0.067595	-0.00782	-0.002770832	-0.000187295
	0.26	0.050887	-0.154640271	1.240775	1.086135	0.05527	-0.01232	-0.004367911	-0.000241416
	0.28	0.051531	-0.394407194	1.15337	0.758963	0.03911	-0.01616	-0.005727295	-0.000223994
	0.3	0.052182	-0.609417051	0.993568	0.384151	0.020046	-0.01906	-0.00675631	-0.000135437
	0.32	0.052842	-0.786173603	0.7714	-0.01477	-0.00078	-0.02083	-0.007380976	5.76226E-06
	0.34	0.05351	-0.913581787	0.50081	-0.41277	-0.02209	-0.02131	-0.007551194	0.000166788
	0.36	0.054187	-0.983644148	0.198784	-0.78486	-0.04253	-0.02044	-0.007244552	0.000308106

	4.46	0.712282	-0.476917021	-1.102716	-1.57963	-1.12514	-0.00113	-0.000401671	0.000451937
	4.48	0.721289	-0.244547061	-1.217569	-1.46212	-1.05461	0.070536	0.024998163	-0.026363244
	4.5	0.730409	0.003173198	-1.255994	-1.25282	-0.91507	0.139535	0.049451453	-0.045251634
	4.52	0.739646	0.250694273	-1.21558	-0.96489	-0.71367	0.201399	0.071375847	-0.050939035
	4.54	0.748999	0.482479189	-1.098863	-0.61638	-0.46167	0.252002	0.089309875	-0.04123177
	4.56	0.75847	0.683978731	-0.913171	-0.22919	-0.17384	0.287835	0.102009152	-0.017732804
	4.58	0.768061	0.842544708	-0.670159	0.172386	0.132403	0.306239	0.108531203	0.014369865
	4.6	0.777773	0.948223883	-0.38508	0.563144	0.437998	0.305595	0.108303204	0.047436614
	4.62	0.787609	0.994382741	-0.07583	0.918553	0.72346	0.285462	0.101167892	0.07319092
	4.64	0.797568	0.978123875	0.23818	1.216304	0.970085	0.246625	0.087404163	0.084789475
	4.66	0.807654	0.900467859	0.537239	1.437707	1.161169	0.191084	0.0677204	0.078634846
	4.68	0.817867	0.766289186	0.802576	1.568865	1.283122	0.121953	0.043220263	0.055456882
	4.7	0.828209	0.584010295	1.017534	1.601545	1.326413	0.043291	0.015342419	0.020350391
	4.72	0.838682	0.36507289	1.168622	1.533695	1.286282	-0.04013	-0.014222609	-0.018294286
	4.74	0.849287	0.123219742	1.246355	1.369575	1.163162	-0.12312	-0.043633759	-0.050753141
	4.76	0.860027	-0.126367948	1.245854	1.119486	0.962788	-0.20037	-0.071012921	-0.068370366
	4.78	0.870902	-0.368023481	1.16715	0.799127	0.695961	-0.26683	-0.09456367	-0.065812612
	4.8	0.881915	-0.586578063	1.015184	0.428606	0.377994	-0.31797	-0.112687824	-0.042595294
	4.82	0.893067	-0.768312951	0.799494	0.031181	0.027847	-0.35015	-0.12409233	-0.003455598
	4.84	0.90436	-0.901820588	0.53362	-0.3682	-0.33299	-0.36083	-0.12787943	0.042582022
	4.86	0.915796	-0.978720658	0.23425	-0.74447	-0.68178	-0.3488	-0.123613929	0.084277823
	4.88	0.927376	-0.994186117	-0.079823	-1.07401	-0.99601	-0.31423	-0.111362697	0.110918427
	4.9	0.939103	-0.947246195	-0.388886	-1.33613	-1.25477	-0.25876	-0.091703192	0.11506606
	4.92	0.950978	-0.840847324	-0.673539	-1.51439	-1.44015	-0.18538	-0.065699652	0.094617251
	4.94	0.963004	-0.681668197	-0.915913	-1.59758	-1.53848	-0.09833	-0.034847667	0.053612328
	4.96	0.975181	-0.479700539	-1.100795	-1.5805	-1.54127	-0.00279	-0.000989852	0.00152563
	4.98	0.987513	-0.247621923	-1.21658	-1.4642	-1.44592	0.095352	0.033792737	-0.04886153
	5	1	-2.33376E-13	-1.256	-1.256	-1.256	0.189918	0.067307152	-0.084537783
							-1.20312	**-0.426385746**	**0.82048948**
									0.138188232
									0.371736777

x(t)=	a(t)*u	0.001096459				sdv=	0.001875308			sdv=	0.049584778
y(t)=s(t)*u											
						average	-1.07849E-05			average	-8.9678E-05
		3.23098E-05				ß=	-0.005751018			ß=	-0.00180858
		0.029467387		process 2 for θ=1				process 2 for θ=0.1			
random no.	**x(t)**	**y(t) process 1**		**part1**	**part 2**	**par1/part2**	**part 3*y(t)**	**part1**	**part 2**	**par1/part2**	**part 3*y(t)**
6.227779198			**theta**								
1.90995E-05	-3.37475E-09	8.3113E-08	0	8.3113E-08	1.4142136	5.87698E-08	4.88453E-15	0.195869533	0.447213595	0.43797759	3.64016E-08
4.392666602	-0.000600798	0.023629596	0.1	0.023629596	1.414411	0.016706315	0.000394763	0.687615454	0.447837423	1.535413117	0.036281191
0.224374824	-1.1261E-05	0.001366542	0.2	0.001366542	1.4142142	0.000966291	1.32048E-06	0.517085438	0.447215683	1.156232792	0.001580041
1.23907942	7.89523E-05	0.007969202	0.3	0.007969202	1.414236	0.005634987	4.49064E-05	0.616795907	0.447284594	1.378978653	0.010989359
2.132167077	0.000378826	0.013585091	0.4	0.013585091	1.4142788	0.009605667	0.000130494	0.650588309	0.447419886	1.454088942	0.019753931
1.276629498	0.000335733	0.007537084	0.5	0.007537084	1.4142336	0.005329448	4.01685E-05	0.613366901	0.447277104	1.371335344	0.01033587
-0.063375183	-1.88081E-05	-0.000319929	0.6	0.000319929	1.4142136	0.000226224	-7.23755E-08	0.447203644	0.44721371	0.999977492	-0.00031992
-0.132195809	-3.54387E-05	-0.000508819	0.7	0.000508819	1.4142137	0.00035979	-1.83068E-07	0.468442571	0.447213885	1.047468754	-0.00053297
-0.140059162	-2.53856E-05	-0.0003325	0.8	0.0003325	1.4142136	0.000235113	-7.81753E-08	0.448930613	0.447213719	1.00383909	-0.00033378
1.422305443	7.91443E-05	0.001010119	0.9	0.001010119	1.4142139	0.000714262	7.21489E-07	0.501692075	0.447214736	1.121814722	0.001133166
1.012542437	-7.95058E-05	-0.001054264	1	0.001054264	1.414214	0.000745477	-7.8593E-07	0.503842662	0.447214838	1.126623312	-0.00118776
-0.808765019	0.000151478	0.002240952	1.1	0.002240952	1.4142153	0.00158459	3.55099E-06	0.543304441	0.44721921	1.214850411	0.002722421
-0.23203831	5.60179E-05	0.001013523	1.2	0.001013523	1.4142139	0.000716669	7.2636E-07	0.50186088	0.447214744	1.122192161	0.001137367
-1.694681164	0.000379599	0.009705939	1.3	0.009705939	1.4142469	0.006862974	6.66116E-05	0.629076926	0.447318908	1.4063276	0.01364973
0.113694488	-1.53984E-05	-0.000768155	1.4	0.000768155	1.4142138	0.000543168	-4.17237E-07	0.488140566	0.447214255	1.091513879	-0.00083845
1.783391028	1.02764E-05	-0.013163166	1.5	0.013163166	1.4142748	0.009307361	-0.000122514	0.648538908	0.447407274	1.449549316	-0.01908066
0.991912846	0.000165439	-0.007490126	1.6	0.007490126	1.4142334	0.005296245	-3.96695E-05	0.612983684	0.447276315	1.370480984	-0.01026508
-0.721871387	-0.000222413	0.005229635	1.7	0.005229635	1.4142232	0.003697885	1.93386E-05	0.591353456	0.447244172	1.322216126	0.006914708
1.099051205	0.000432103	-0.007109163	1.8	0.007109163	1.4142314	0.005026874	-3.57369E-05	0.609792177	0.447270098	1.36336451	-0.00969238
-1.641214349	-0.000652899	0.00863346	1.9	0.00863346	1.4142399	0.006104664	5.27044E-05	0.621753858	0.447296922	1.390024896	0.012000724
0.84764815	0.000268812	-0.003124774	2	0.003124774	1.414217	0.002209544	-6.90433E-06	0.561670827	0.447224512	1.255903494	-0.00392442
0.489947657	8.21118E-05	-0.000900244	2.1	0.000900244	1.4142138	0.000636568	-5.73067E-07	0.495947861	0.447214502	1.108970883	-0.00099834
-0.662747154	1.05628E-05	-0.00011644	2.2	0.00011644	1.4142136	8.23355E-05	-9.58715E-09	0.404212959	0.447213611	0.903847622	-0.00010524
0.101253468	-1.90531E-05	0.000225679	2.3	0.000225679	1.4142136	0.000159579	3.60137E-08	0.431866136	0.447213652	0.965681913	0.000217934
-0.621877111	0.000189264	-0.002607056	2.4	0.002607056	1.414216	0.001843464	-4.80601E-06	0.551588296	0.447221194	1.233367969	-0.00321546
0.369561814	-0.000122549	0.002194622	2.5	0.002194622	1.4142153	0.00155183	3.40568E-06	0.542170612	0.44721898	1.212315746	0.002660575

		ß=	-0.00542483			ß=	-0.00720688			ß=	-0.00746818
process 2 for θ=0.3				process 2 for θ=0.5				process 2 for θ=0.7			
part1	**part 2**	**par1/part2**	**part 3*y(t)**	**part1**	**part 2**	**par1/part2**	**part 3*y(t)**	**part1**	**part 2**	**par1/part2**	**part 3*y(t)**
0.00751451	0.774596669	0.009701191	8.06295E-10	0.000288293	1	0.000288293	2.39609E-11	1.10603E-05	1.183215957	9.34769E-06	7.76914E-13
0.32511491	0.774957004	0.419526384	0.009913239	0.15371921	1.00027914	0.153676313	0.003631309	0.07268075	1.183451882	0.0614142	0.001451193
0.138256934	0.774597875	0.178488657	0.000243912	0.036966773	1.000000934	0.036966738	5.05166E-05	0.009884078	1.183216746	0.008353565	1.14155E-05
0.234652102	0.774637663	0.302918531	0.002414019	0.089270387	1.000031754	0.089267552	0.000711391	0.033961775	1.183242793	0.028702288	0.000228734
0.275371357	0.77471579	0.355448231	0.004828797	0.116555098	1.000092273	0.116544344	0.001583266	0.049333711	1.183293943	0.041691848	0.000566388
0.230760255	0.774633338	0.297896106	0.002245268	0.086816382	1.000028403	0.086813916	0.000654324	0.032661969	1.183239962	0.027603842	0.000208052
0.089436748	0.774596735	0.115462336	-3.694E-05	0.017886554	1.000000051	0.017886553	-5.7224E-06	0.003577152	1.183216	0.003023245	-9.6722E-07
0.102794308	0.774596836	0.132706852	-6.7524E-05	0.022557023	1.000000129	0.02255702	-1.1477E-05	0.004949878	1.183216066	0.00418341	-2.1286E-06
0.09047689	0.774596741	0.116805152	-3.8838E-05	0.018234594	1.000000055	0.018234593	-6.063E-06	0.003674976	1.183216003	0.003105922	-1.0327E-06
0.126273356	0.774597328	0.163018063	0.000164668	0.031782364	1.00000051	0.031782348	3.21039E-05	0.00799946	1.183216388	0.006760775	6.82919E-06
0.127904202	0.774597387	0.165123462	-0.00017408	0.032469432	1.000000556	0.032469414	-3.4231E-05	0.008242606	1.183216426	0.006966271	-7.3443E-06
0.160372451	0.774599911	0.207039077	0.000463965	0.047338694	1.000002511	0.047338576	0.000106083	0.013973422	1.183218079	0.011809676	2.64649E-05
0.126400861	0.774597332	0.16318267	0.000165389	0.03183587	1.000000514	0.031835853	3.22664E-05	0.00801832	1.183216391	0.006776715	6.86835E-06
0.248949505	0.774657476	0.321367202	0.00311917	0.098518724	1.000047102	0.098514084	0.000956172	0.038987581	1.183255765	0.032949412	0.000319805
0.116314726	0.77459705	0.150161592	-0.00011535	0.027715614	1.000000295	0.027715606	-2.129E-05	0.00660411	1.183216206	0.005581491	-4.2875E-06
0.272777226	0.774708506	0.352103047	-0.00463479	0.114730842	1.000086631	0.114720904	-0.00151009	0.048256104	1.183289174	0.040781328	-0.00053681
0.230328005	0.774632882	0.297338275	-0.0022271	0.086545517	1.000028051	0.086543089	-0.00064822	0.032519391	1.183239664	0.027483351	-0.00020585
0.206795659	0.774614323	0.266965963	0.001396135	0.072316216	1.000013674	0.072315228	0.000378182	0.025288902	1.183227514	0.021372814	0.000111772
0.226749087	0.774629292	0.292719484	-0.00208099	0.084315854	1.00002527	0.084313724	-0.0005994	0.031352555	1.183237314	0.026497267	-0.00018837
0.240356275	0.774644781	0.310279345	0.002678784	0.092916414	1.000037268	0.092912952	0.00080216	0.035919429	1.183247454	0.03035665	0.000262083
0.177192609	0.774602972	0.228752813	-0.0007148	0.055899682	1.000004882	0.055899409	-0.00017467	0.017634903	1.183220083	0.014904161	-4.6572E-05
0.121985459	0.774597192	0.157482444	-0.00014177	0.030004066	1.000000405	0.030004054	-2.7011E-05	0.007379928	1.183216299	0.006237176	-5.615E-06

sdv=	0.017398821	sdv=	0.02924365	sdv=	0.027611338	sdv=	0.0248945	sdv=	0.02177563	sdv=	0.01874822	
average	-0.2926594	average	-0.0497161	average	-0.14050266	average	-0.21101004	average	-0.2580633	average	-0.2850351	
ß=	-16.8206459	ß=	-1.7000663	ß=	-5.08858556	ß=	-8.47616926	ß=	-11.851015	ß=	-15.20331	
process 3 for 1/m =1		process 3 for 1/m =0.1		process 3 for 1/m =0.3		process 3 for 1/m =0.5		process 3 for 1/m =0.7		process 3 for 1/m =0.9		
part 1	**part-part2**	**part 1**	**part-part2**	**part 1**	**part-part2**	**part 1**	**part-part2**	**part 1**	**part-part2**	**part 1**	**part-part2**	
0.707109	-0.29289063	9.950273	-0.049727	3.192742	-0.14059176	1.788845	-0.21115536	1.17033	-0.2582412	0.825885	-0.2852264	-5659.537867
0.721159	-0.27884126	9.9268474	-0.0731526	3.170671	-0.16266215	1.768988	-0.23101161	1.152997	-0.2755747	0.810989	-0.3001224	-7311.381311
0.707914	-0.29208569	9.9489169	-0.0510831	3.191461	-0.14187223	1.78769	-0.21230966	1.169321	-0.2592506	0.825016	-0.2860952	-19924.97369
0.711817	-0.2881833	9.9423665	-0.0576335	3.185281	-0.14805217	1.782123	-0.21787672	1.164456	-0.2641159	0.820831	-0.2902804	15694.03002
0.715153	-0.2848472	9.9367985	-0.0632015	3.180034	-0.1532991	1.777402	-0.22259813	1.160333	-0.2682381	0.817288	-0.2938235	5628.361802
0.711561	-0.28843935	9.9427951	-0.0572049	3.185685	-0.14764809	1.782487	-0.21751291	1.164773	-0.2637981	0.821104	-0.2900072	3802.511088
0.706921	-0.29307901	9.9505907	-0.0494093	3.193042	-0.14029181	1.789115	-0.21088492	1.170567	-0.2580047	0.826088	-0.2850228	3369.566757
0.70681	-0.29319018	9.9507782	-0.0492218	3.193219	-0.14011475	1.789275	-0.21072528	1.170706	-0.257865	0.826209	-0.2849026	3730.263572
0.706914	-0.29308641	9.9506031	-0.0493969	3.193053	-0.14028003	1.789126	-0.2108743	1.170576	-0.2579954	0.826096	-0.2850148	5517.267093
0.707704	-0.29229573	9.9492706	-0.0507294	3.191795	-0.14153829	1.787991	-0.21200865	1.169584	-0.2589874	0.825242	-0.2858687	17971.03186
0.706489	-0.29351109	9.9513196	-0.0486804	3.19373	-0.13960341	1.789736	-0.2102642	1.17111	-0.2574617	0.826556	-0.2845554	-12735.44752
0.70843	-0.29157011	9.9480491	-0.0519509	3.190642	-0.14269135	1.786952	-0.21304792	1.168675	-0.2598961	0.82446	-0.2866507	-5339.169275
0.707706	-0.29229373	9.9492672	-0.0507328	3.191792	-0.14154147	1.787988	-0.21201152	1.169581	-0.2589899	0.82524	-0.2858708	-4142.219589
0.712847	-0.28715326	9.9406443	-0.0593557	3.183658	-0.14967573	1.780662	-0.21933817	1.163179	-0.2653923	0.819733	-0.2913778	-4464.39861
0.706657	-0.29334278	9.9510356	-0.0489644	3.193462	-0.13987164	1.789494	-0.21050607	1.170898	-0.2576733	0.826374	-0.2847375	-7383.530034
0.699402	-0.30059804	9.9633467	-0.0366533	3.205103	-0.12823049	1.800002	-0.19999757	1.180099	-0.2484725	0.834301	-0.2768099	173542.8974
0.702713	-0.29728669	9.9577101	-0.0422899	3.19977	-0.13356375	1.795185	-0.20481481	1.175879	-0.2526925	0.830663	-0.2804477	5995.638997
0.710195	-0.28980508	9.9450839	-0.0549161	3.187844	-0.14548945	1.784431	-0.21556895	1.166472	-0.2620997	0.822565	-0.2885465	3245.638475
0.702936	-0.29706376	9.9573317	-0.0426683	3.199412	-0.13392158	1.794862	-0.20513784	1.175596	-0.2529754	0.83042	-0.2806914	2543.494657
0.71221	-0.28778951	9.9417078	-0.0582922	3.18466	-0.14867324	1.781564	-0.21843583	1.163967	-0.2646043	0.820411	-0.2907003	2513.735365
0.705272	-0.29472797	9.9533751	-0.0466249	3.195672	-0.13766161	1.791487	-0.20851286	1.172642	-0.2559295	0.827875	-0.2832361	3153.31372

			average=	4.812E-05			-0.000252393			-0.7382023
			sdv=	0.000313481			0.001644232			4.809070065
||a(t)||	a(t)/||a(t)	U* pr0cess 1	f*(t) process1	x*(t) process 1	U* process 2	f*(t) proceess2	x*(t) process 2	U* process 3	f*(t) proceess3	x*(t) process 3
0.371736777	45.502823	0.049891994	0.000217109	-8.81556E-06	-0.261687562	-0.001138754	4.62383E-05	-765.386872	-3.330640021	0.135238405
0.371736777	45.514303	0.049904581	0.000268453	-6.8256E-06	-0.261753585	-0.001408058	3.58008E-05	-765.5799764	-4.118305996	0.104710717
0.371736777	45.538755	0.049931392	0.000304104	-2.50597E-06	-0.261894207	-0.001595052	1.3144E-05	-765.9912717	-4.665226898	0.038443778
0.371736777	45.569986	0.049965636	0.000321357	3.18374E-06	-0.26207382	-0.001685541	-1.66989E-05	-766.5166037	-4.9298903	-0.048841286
0.371736777	45.600164	0.049998724	0.000318567	8.88335E-06	-0.262247371	-0.001670908	-4.65939E-05	-767.0242097	-4.887090589	-0.136278412
0.371736777	45.622057	0.050022729	0.000295329	1.31552E-05	-0.26237328	-0.001549024	-6.9E-05	-767.3924689	-4.530603158	-0.201812027
0.371736777	45.630563	0.050032055	0.00025257	1.48482E-05	-0.262422195	-0.001324752	-7.78801E-05	-767.5355368	-3.874651075	-0.227784636
0.371736777	45.623345	0.050024142	0.000192542	1.34104E-05	-0.262380688	-0.001009899	-7.03384E-05	-767.4141361	-2.953763119	-0.205726518
0.371736777	45.601094	0.049999744	0.000118699	9.06241E-06	-0.262252721	-0.000622588	-4.75331E-05	-767.0398552	-1.820952516	-0.139025326
0.371736777	45.567808	0.049963247	3.54838E-05	2.78021E-06	-0.262061291	-0.000186115	-1.45824E-05	-766.4799595	-0.544352648	-0.04265086
0.371736777	45.530821	0.049922693	-5.19797E-05	-3.91998E-06	-0.261848581	0.000272638	2.05606E-05	-765.8578226	0.797414809	0.060135918
0.371736777	45.499752	0.049888627	-0.000138233	-9.34389E-06	-0.261669901	0.000725043	4.90095E-05	-765.3352175	2.120615303	0.143343501
0.371736777	45.483933	0.049871281	-0.000217833	-1.20397E-05	-0.261578924	0.001142553	6.31495E-05	-765.0691264	3.341753567	0.184700282
0.371736777	45.489051	0.049876894	-0.00028566	-1.11721E-05	-0.26160836	0.001498308	5.85988E-05	-765.1552227	4.382269718	0.171390436
0.371736777	45.514685	0.049905	-0.000337174	-6.75896E-06	-0.261755782	0.001768503	3.54513E-05	-765.5864035	5.172539417	0.103688398
0.371736777	45.554229	0.049948358	-0.000368668	2.87816E-07	-0.261983197	0.001933692	-1.50962E-06	-766.2515507	5.655684177	-0.004415344
0.371736777	45.597328	0.049995614	-0.000377527	8.33866E-06	-0.262231061	0.001980158	-4.3737E-05	-766.9765042	5.791588484	-0.127922396
0.371736777	45.633394	0.05003516	-0.000362482	1.54161E-05	-0.262438478	0.001901249	-8.08588E-05	-767.5831611	5.560796504	-0.236496815
0.371736777	45.654307	0.05005809	-0.000323799	1.96808E-05	-0.262558749	0.001698349	-0.000103228	-767.9349311	4.967352549	-0.301921189
0.371736777	45.655434	0.050059326	-0.000263333	1.99143E-05	-0.262565232	0.001381201	-0.000104452	-767.9538914	4.039752191	-0.305503078
0.371736777	45.63564	0.050037623	-0.000184459	1.58683E-05	-0.262451397	0.000967502	-8.32303E-05	-767.6209464	2.829761773	-0.243433104
0.371736777	45.597538	0.049995845	-9.18638E-05	8.37895E-06	-0.262232269	0.000481833	-4.39483E-05	-766.9800388	1.409271246	-0.128540492
0.371736777	45.548258	0.049941811	8.77443E-06	-7.95971E-07	-0.261948858	-4.60226E-05	4.17493E-06	-766.1511151	-0.134607386	0.012210891
0.371736777	45.499498	0.049888347	0.000111194	-9.38763E-06	-0.261668437	-0.000583221	4.92389E-05	-765.3309344	-1.705810824	0.144014427

Example 2 Method-1

						F(t)=s(t)^T*u X(t)=int(0-t)[Ft*h(t-τ)]dτ			
calculate si(τ)						sigma=	0.3544		
wf=	12.566	5	t=						
£f=	0.05								
τ	part 1	part2	part3	hf(t-τ)	s(t)	normalized s(t)	random no.	F(t)	Hs(t-τ)
0	0.000373	0.121633	1.252537	-0.00042	0.001192	0.000422339	6.227779198	0.002630236	-0.00124
0.02	0.000377	3.230501	1.188207	0.00077	0.001173	0.000415654	1.90995E-05	7.93879E-09	-0.00183
0.04	0.000382	6.136447	1.049408	0.001943	0.001079	0.000382296	4.392666602	0.001679299	-0.0019
0.06	0.000387	8.656936	0.844838	0.003022	0.000913	0.000323714	0.224374824	7.26332E-05	-0.0014
0.08	0.000392	10.63364	0.58732	0.003935	0.000686	0.000243003	1.23907942	0.0003011	-0.00044
0.1	0.000397	11.94241	0.292992	0.004621	0.000408	0.000144748	2.132167077	0.000308626	0.00071
0.12	0.000402	12.50102	-0.0197	0.005029	9.81E-05	3.47676E-05	1.276629498	4.43853E-05	0.001705
0.14	0.000407	12.27438	-0.33116	0.005128	-0.00023	-8.02221E-05	-0.063375183	5.08409E-06	0.002239
0.16	0.000412	11.27674	-0.62186	0.004901	-0.00054	-0.000193017	-0.132195809	2.5516E-05	0.00214
0.18	0.000417	9.570759	-0.87358	0.004357	-0.00084	-0.000296368	-0.140059162	4.1509E-05	0.001365
0.2	0.000422	7.263595	-1.07056	0.00352	-0.00108	-0.000383446	1.422305443	-0.000545378	0.000131
0.22	0.000428	4.500174	-1.20044	0.002438	-0.00126	-0.000448286	1.012542437	-0.000453908	-0.00121
0.24	0.000433	1.454076	-1.25509	0.001173	-0.00137	-0.000486187	-0.808765019	0.000393211	-0.00226
0.26	0.000439	-1.68336	-1.23107	-0.0002	-0.00139	-0.00049405	-0.23203831	0.000114639	-0.00269
0.28	0.000444	-4.71505	-1.1299	-0.00159	-0.00133	-0.000470611	-1.694681164	0.000797535	-0.00232
0.3	0.00045	-7.45057	-0.95791	-0.00292	-0.00118	-0.000416568	0.113694488	-4.73614E-05	-0.00122
0.32	0.000455	-9.71809	-0.72589	-0.0041	-0.00094	-0.00033459	1.783391028	-0.000596705	0.000307
0.34	0.000461	-11.3752	-0.44837	-0.00504	-0.00065	-0.000229197	0.991912846	-0.000227343	0.001827

	ß=	||u*||=	112.0299985	10.58442245
£f=	0.05			
wo=	27.53443959			
wd=	27.5	xo=	0.33	
H(t)=	50	ß=	10.58442245	#DIV/0!
||a(t)||=	0.000972061	0.031178		
a(t)	ά	u*	f*(t)	x*(t)
-5.24642E-07	-1.6827E-05	-0.00018	-7.5222E-08	9.3443E-11
-7.62424E-07	-2.4454E-05	-0.00026	-1.07584E-07	1.97339E-10
-7.27235E-07	-2.3325E-05	-0.00025	-9.43832E-08	1.79544E-10
-4.51877E-07	-1.4494E-05	-0.00015	-4.96594E-08	6.93204E-11
-1.06088E-07	-3.4027E-06	-3.6E-05	-8.75183E-09	3.8208E-12
1.0274E-07	3.29528E-06	3.49E-05	5.04859E-09	3.58342E-12
5.92889E-08	1.90163E-06	2.01E-05	6.99791E-10	1.19335E-12
-1.79602E-07	-5.7606E-06	-6.1E-05	4.89132E-09	1.09507E-11
-4.13012E-07	-1.3247E-05	-0.00014	2.70631E-08	5.79089E-11
-4.04503E-07	-1.2974E-05	-0.00014	4.06981E-08	5.55475E-11
-5.03452E-08	-1.6148E-06	-1.7E-05	6.55365E-09	8.60473E-13
5.43291E-07	1.74255E-05	0.000184	-8.26814E-08	1.00204E-10
1.10011E-06	3.5285E-05	0.000373	-1.81577E-07	4.10861E-10
1.32662E-06	4.255E-05	0.00045	-2.22504E-07	5.97466E-10
1.08963E-06	3.49489E-05	0.00037	-1.74086E-07	4.03071E-10
5.08573E-07	1.6312E-05	0.000173	-7.19216E-08	8.78065E-11
-1.02586E-07	-3.2903E-06	-3.5E-05	1.16525E-08	3.57267E-12
-4.18797E-07	-1.3432E-05	-0.00014	3.25861E-08	5.95425E-11

-0.035718238	-0.000195889	-0.44644	-0.002448399	-0.07852995	-0.83119	-0.004558514	0.002035095
-0.035718238	-0.000147911	-0.86767	-0.003593058	-0.11524378	-1.21979	-0.005051194	0.004382772
-0.035718238	-8.92822E-05	-1.04899	-0.002622079	-0.08410059	-0.89016	-0.002225056	0.00233406
-0.035718238	-2.34719E-05	-0.92172	-0.000605698	-0.01942716	-0.20563	-0.000135125	0.000124547
-0.035718238	4.55062E-05	-0.50709	0.000646051	0.020721453	0.219325	-0.000279427	0.000141695
-0.035718238	0.000113334	0.085109	-0.00027005	-0.00866158	-0.09168	0.000290894	2.47576E-05
-0.035718238	0.000175658	0.684924	-0.003368366	-0.108037	-1.14351	0.005623632	0.003851758
-0.035718238	0.000228363	1.110469	-0.007099737	-0.22771701	-2.41025	0.015409857	0.017112162
-0.035718238	0.000267845	1.222545	-0.009167672	-0.29404397	-3.11229	0.02333854	0.028532413
-0.035718238	0.000291251	0.969363	-0.007904316	-0.25352308	-2.6834	0.021880762	0.021210403
-0.035718238	0.00029668	0.407226	-0.00338247	-0.10848937	-1.1483	0.009537893	0.003884081
-0.035718238	0.000283329	-0.31047	0.00246276	0.078990567	0.83607	-0.006631976	0.002059038
-0.035718238	0.000251572	-0.97438	0.006862791	0.220117195	2.329813	-0.016409438	0.015989021
-0.035718238	0.000202968	-1.37966	0.007839883	0.251456458	2.661521	-0.015124017	0.020866016
-0.035718238	0.000140187	-1.38851	0.005449617	0.174791062	1.850062	-0.007261134	0.010082132
-0.035718238	6.68732E-05	-0.97581	0.001826959	0.058597902	0.620225	-0.001161212	0.001133126
-0.035718238	-1.25664E-05	-0.24318	-8.55575E-05	-0.00274417	-0.02905	-1.02188E-05	2.48506E-06
-0.035718238	-9.3222E-05	0.604783	0.00157844	0.050626895	0.535856	0.001398547	0.000845817
-0.035718238	-0.000169981	1.31687	0.006266924	0.201005372	2.127526	0.010124795	0.013333043
-0.035718238	-0.000237851	1.668954	0.011113725	0.356461691	3.772941	0.025124376	0.041931429
-0.035718238	-0.000292278	1.533663	0.012549769	0.402521384	4.260456	0.034862777	0.053467743
-0.035718238	-0.00032945	0.924562	0.008527761	0.273519464	2.895046	0.026702647	0.024688256
-0.035718238	-0.000137109	9.33E-13	3.57987E-15	1.14821E-13	1.22E-12	4.66514E-15	4.35065E-27
					13.27077	**0.193505523**	**0.33**
					112.03	**0.005477912**	**0.008231123**
					10.58442	**0.07401292**	**0.090725537**

Alternative Approach

q1	**q1**	**q1**	**q2**	**q2**	**q2**	**q3**	**q3**	**q3**
0.311241	0.311241	0.311241	0.30468317	0.30468317	0.30468317	0.10764274	0.10764274	0.10764274
-0.07339	-0.07339	-0.07339	-0.0658898	-0.0658898	-0.0658898	-0.1066948	-0.1066948	-0.1066948
-0.14579	-0.14579	-0.14579	-0.1105133	-0.1105133	-0.1105133	0.00880283	0.00880283	0.00880283
0.003126	0.003126	0.003126	-0.0044363	-0.0044363	-0.0044363	0.00422281	0.00422281	0.00422281
-0.03018	-0.03018	-0.03018	-0.0164255	-0.0164255	-0.0164255	0.07010703	0.07010703	0.07010703
-0.14993	-0.14993	-0.14993	-0.1520238	-0.1520238	-0.1520238	-0.0171117	-0.0171117	-0.0171117
-0.04231	-0.04231	-0.04231	-0.0232244	-0.0232244	-0.0232244	0.04755298	0.04755298	0.04755298
-0.14944	-0.14944	-0.14944	-0.1354517	-0.1354517	-0.1354517	-0.0667716	-0.0667716	-0.0667716
0.188706	0.188706	0.188706	0.2118413	0.2118413	0.2118413	0.14008979	0.14008979	0.14008979
-0.07911	-0.07911	-0.07911	-0.0388018	-0.0388018	-0.0388018	0.08460288	0.08460288	0.08460288
0.190298	0.190298	0.190298	0.18246026	0.18246026	0.18246026	0.02343171	0.02343171	0.02343171
0.134716	0.134716	0.134716	0.16299902	0.16299902	0.16299902	0.15503317	0.15503317	0.15503317
0.212892	0.212892	0.212892	0.22234051	0.22234051	0.22234051	0.02086364	0.02086364	0.02086364
-0.03787	-0.03787	-0.03787	-0.0225838	-0.0225838	-0.0225838	0.08329657	0.08329657	0.08329657
0.004469	0.004469	0.004469	-0.0241297	-0.0241297	-0.0241297	-0.0586752	-0.0586752	-0.0586752
0.089338	0.089338	0.089338	0.07203855	0.07203855	0.07203855	-0.0582809	-0.0582809	-0.0582809
0.095325	0.095325	0.095325	0.08568323	0.08568323	0.08568323	-0.0744723	-0.0744723	-0.0744723
-0.30117	-0.30117	-0.30117	-0.2928532	-0.2928532	-0.2928532	-0.0894548	-0.0894548	-0.0894548

						m1				
q4	q4	q4	q5	q5	q5	q1	q2	q3	q4	q5
0.00048702	0.00048702	0.00048702	0.01381748	0.01381748	0.01381748	0.311241	0.304683	0.107643	0.000487	0.013817
-0.0251099	-0.0251099	-0.0251099	-0.0257049	-0.0257049	-0.0257049	-0.07339	-0.06589	-0.10669	-0.02511	-0.0257
-0.0331878	-0.0331878	-0.0331878	-0.0336924	-0.0336924	-0.0336924	-0.14579	-0.11051	0.008803	-0.03319	-0.03369
-0.0040433	-0.0040433	-0.0040433	-0.0091385	-0.0091385	-0.0091385	0.003126	-0.00444	0.004223	-0.00404	-0.00914
0.02416468	0.02416468	0.02416468	0.02258475	0.02258475	0.02258475	-0.03018	-0.01643	0.070107	0.024165	0.022585
-0.00738	-0.00738	-0.00738	-0.0073853	-0.0073853	-0.0073853	-0.14993	-0.15202	-0.01711	-0.00738	-0.00739
0.00420061	0.00420061	0.00420061	0.00611776	0.00611776	0.00611776	-0.04231	-0.02322	0.047553	0.004201	0.006118
0.02912743	0.02912743	0.02912743	0.03010808	0.03010808	0.03010808	-0.14944	-0.13545	-0.06677	0.029127	0.030108
0.01186746	0.01186746	0.01186746	0.00234649	0.00234649	0.00234649	0.188706	0.211841	0.14009	0.011867	0.002346
-0.0523785	-0.0523785	-0.0523785	-0.0473572	-0.0473572	-0.0473572	-0.07911	-0.0388	0.084603	-0.05238	-0.04736
0.00055945	0.00055945	0.00055945	-0.0065731	-0.0065731	-0.0065731	0.190298	0.18246	0.023432	0.000559	-0.00657
0.02278051	0.02278051	0.02278051	0.02283523	0.02283523	0.02283523	0.134716	0.162999	0.155033	0.022781	0.022835
-0.0083537	-0.0083537	-0.0083537	-0.0171939	-0.0171939	-0.0171939	0.212892	0.222341	0.020864	-0.00835	-0.01719
0.07266209	0.07266209	0.07266209	0.06942735	0.06942735	0.06942735	-0.03787	-0.02258	0.083297	0.072662	0.069427
0.02272085	0.02272085	0.02272085	0.03147123	0.03147123	0.03147123	0.004469	-0.02413	-0.05868	0.022721	0.031471
-0.0145442	-0.0145442	-0.0145442	-0.0120841	-0.0120841	-0.0120841	0.089338	0.072039	-0.05828	-0.01454	-0.01208
0.00616026	0.00616026	0.00616026	0.00383695	0.00383695	0.00383695	0.095325	0.085683	-0.07447	0.00616	0.003837
-0.0763478	-0.0763478	-0.0763478	-0.0827749	-0.0827749	-0.0827749	-0.30117	-0.29285	-0.08945	-0.07635	-0.08277

	m2					p			
q1	q2	q3	q4	q5	q1	q2	q3	q4	q5
0.311241	0.304683	0.107643	0.000487	0.013817	0.311241	0.304683	0.107643	0.000487	0.013817
-0.07339	-0.06589	-0.10669	-0.02511	-0.0257	-0.07339	-0.06589	-0.10669	-0.02511	-0.0257
-0.14579	-0.11051	0.008803	-0.03319	-0.03369	-0.14579	-0.11051	0.008803	-0.03319	-0.03369
0.003126	-0.00444	0.004223	-0.00404	-0.00914	0.003126	-0.00444	0.004223	-0.00404	-0.00914
-0.03018	-0.01643	0.070107	0.024165	0.022585	-0.03018	-0.01643	0.070107	0.024165	0.022585
-0.14993	-0.15202	-0.01711	-0.00738	-0.00739	-0.14993	-0.15202	-0.01711	-0.00738	-0.00739
-0.04231	-0.02322	0.047553	0.004201	0.006118	-0.04231	-0.02322	0.047553	0.004201	0.006118
-0.14944	-0.13545	-0.06677	0.029127	0.030108	-0.14944	-0.13545	-0.06677	0.029127	0.030108
0.188706	0.211841	0.14009	0.011867	0.002346	0.188706	0.211841	0.14009	0.011867	0.002346
-0.07911	-0.0388	0.084603	-0.05238	-0.04736	-0.07911	-0.0388	0.084603	-0.05238	-0.04736
0.190298	0.18246	0.023432	0.000559	-0.00657	0.190298	0.18246	0.023432	0.000559	-0.00657
0.134716	0.162999	0.155033	0.022781	0.022835	0.134716	0.162999	0.155033	0.022781	0.022835
0.212892	0.222341	0.020864	-0.00835	-0.01719	0.212892	0.222341	0.020864	-0.00835	-0.01719
-0.03787	-0.02258	0.083297	0.072662	0.069427	-0.03787	-0.02258	0.083297	0.072662	0.069427
0.004469	-0.02413	-0.05868	0.022721	0.031471	0.004469	-0.02413	-0.05868	0.022721	0.031471
0.089338	0.072039	-0.05828	-0.01454	-0.01208	0.089338	0.072039	-0.05828	-0.01454	-0.01208
0.095325	0.085683	-0.07447	0.00616	0.003837	0.095325	0.085683	-0.07447	0.00616	0.003837
-0.30117	-0.29285	-0.08945	-0.07635	-0.08277	-0.30117	-0.29285	-0.08945	-0.07635	-0.08277

m1	m2	p	G(x)	
0.109594	2.370973	0.459649	-1.090892305	TRUE
0.118209	-0.52743	0.19936	0.736175718	FALSE
0.211434	-1.17025	0.073979	-0.058968913	TRUE
0.062982	0.043346	0.262945	1.238828889	FALSE
0.139676	0.036303	0.187932	1.122893396	FALSE
0.103786	-1.08304	0.165623	0.158457826	FALSE
0.166286	-0.16585	0.174831	1.020529038	FALSE
0.146052	-0.67239	0.134977	0.525071394	FALSE
0.132099	1.519859	0.279725	-0.204124479	TRUE
0.245412	-0.86632	0.100124	0.268065892	FALSE
0.043829	1.468	0.372799	-0.143280137	TRUE
0.180877	1.236417	0.24509	0.05729615	FALSE
0.086236	1.618684	0.34293	-0.291253282	TRUE
0.136947	0.361255	0.176254	0.870376633	FALSE
0.040214	0.318952	0.340528	1.008369622	FALSE
0.049139	0.66908	0.356466	0.657528522	FALSE
0.0527	0.908941	0.33766	0.419477046	FALSE
0.123032	-2.70556	0.048221	-1.619785928	TRUE

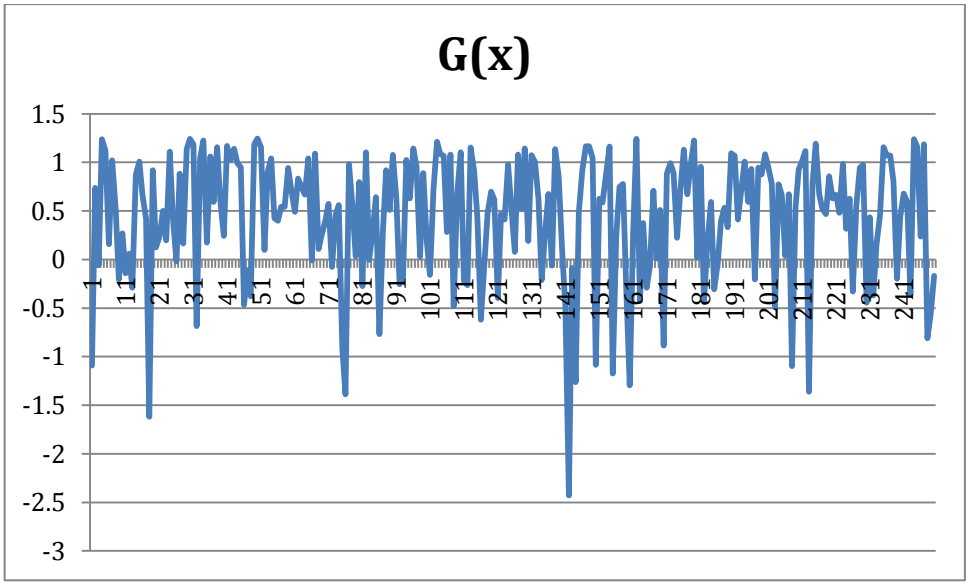

Fig-30 Realization of Limit State Function in U-space

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