“Consolidation of Thin Clay Lamina Using Laurent Transform”

Submitted in Partial Fulfilment for the Award of the Degree of

MASTER OF TECHNOLOGY

IN

CIVIL ENGINEERING

With Specialisation in

GEOTECHNICAL ENGINEERING

By

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2011-2013

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CERTIFICATE

This is to certify that the project report entitled ***"Consolidation of thin clay lamina using Laurent Transform"*** is a bonafide record of work carried out by **Swapnil Mishra (2K11/GTE/11)** under my guidance and supervision, during the session 2013 in partial fulfilment of the requirement for the degree of Master of Technology (Geotechnical Engineering) from Delhi Technological University, Delhi. This is to certify that the above statement made by candidate is correct to the best of my knowledge.

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ACKNOWLEDGEMENT

It is distinct pleasure to express my deep sense of gratitude and indebtedness to my supervisor and H.O.D **Prof. A. Trivedi**, Department of Civil Engineering, Delhi Technological University (Formerly Delhi College of Engineering), for his invaluable guidance, encouragement and patient reviews. His continuous inspiration only has made me complete this major project. Without his help and guidance, this major project would have been impossible. He remained a pillar of help throughout the project.

I express my sincere gratitude to the faculty and non teaching staff of Civil and Environmental engineering department and the library of Delhi Technological University, (Formerly Delhi College of Engineering) for providing the relevant information and help when needed during the course of my project work.

I am deeply thankful towards all the lab assistants of **'Swati constructions'** who have helped me to conduct the experiments. Also I am deeply thankful towards the lab assistants of my college who helped me a lot for conducting experiments here.

I am also thankful to my friend Pankaj Kumar Dahayat and senior Lucky Krishnia for their unconditional support and motivation during this project work.

SWAPNIL MISHRA
(2K11/GTE/11)

ABSTRACT

This work evaluates the rate of consolidation by Laurent series using transformations. A clear comparison is made between Fourier and Laurent outputs for variation of degree of consolidation with time factor experimentally and verified theoretically. Laurent series has been observed to be consistent for explaining consolidation of clay layer with low thickness. As the Laurent series is a type of convergent series therefore all the variation in sample thickness & pore water pressure converges to a single value. Curve obtained by Laurent series continuous in nature. With the help of Laurent series process of consolidation can be studied even at microscopic level and for all kind of soils.

This analysis can be well supported & explained by conversion of nature of plots from Fourier transformation to Laurent transformation.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVATIONS AND NOTATIONS

S.NO.	SYMBOL	NOMENCLATURE
1	u	Pore water pressure
2	C_v	Coefficient of consolidation
3	M	An integer
4	Δσ	Effective pressure increment
5	Δρ	Consolidating pressure
6	H	Depth of soil strata
7	m_v	Coefficient of volume compressibility
8	U	Degree of consolidation
9	T_v	Time factor
10	z_0	Initial depth to be consolidated
11	W_1	Initial water content to blow n_1
12	W_2	Final water content for blow n_2
13	W_L	Liquid limit
14	I_f	Flow Index
15	G	Specific gravity.
16	C_u	Coefficient of uniformity
17	C_c	Coefficient of curvature
18	D_{10}	Effective diameter
19	D_{30}	Diameter corresponding to 30% finer.
20	D_{60}	Diameter corresponding to 60%.
21	MDD	Maximum dry density.
22	OMC	Optimum moisture content.
23	e	Void ratio
24	Q	Discharge per unit volume
25	V	Velocity of flow.
26	K	Coefficient of permeability.
27	C_c	Coefficient of compression.

S.NO.	SYMBOL	NOMENCLATURE
28	t_c	Thickness of clay layer.
29	t_s	Thickness of sand layer.
30	$S_{1.00}$	Sample set with 100% clay.
31	$S_{0.75}$	Sample set with 75% thick clay layer and 25% thick sand layer.
32	$S_{0.50}$	Sample set with 50% thick clay layer and 50% thick sand layer.
33	a_v	Coefficient of Compressibility.

Dedication

I dedicate this thesis to

my family, my teachers and my friends for

supporting me all the way and doing all the

wonderful things for me.

CHAPTER – 1

INTRODUCTION

1.1 MOTIVATION

In 1925, Karl Terzaghi proposed the one-dimensional consolidation theory, where the excess pore water pressure (u), depth within the clay layer (z) and time (t) are related by the following governing differential equation.

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \quad (1)$$

Where C_v is the coefficient of consolidation. When the initial excess pore water pressure is uniform, for double- or single- drained clay layers, the average degree of consolidation (U) is related to the dimensionless time factor (T_v) by

$$U(\%) = \sum_{m=0}^{\infty} \left(\frac{2}{M^2}\right) . e^{-m^2.t} \quad (2)$$

Where $M = \frac{(2m+1)\pi}{2}$ (3)

And $T_v = \frac{C_v.t}{H^2}$ (4)

With H being the maximum length of drainage path. When applying Terzaghi's consolidation theory and the above expressions, it is implied that the pressure at the ground level causing consolidation is applied instantaneously. In reality, building or embankment loads are never placed on the ground instantaneously; they are applied gradually over a certain time period, herein referred to as construction time t_0, as shown in Figure 1. Here, the entire pressure q_0 is applied over the construction time t_0 which can be several months, depending on the nature of the project. Therefore, Terzaghi's solutions, developed for instantaneous loading, will not produce reliable estimates of the consolidation settlements.

The objective of this technical note is to propose a simple analytical solution to one-dimensional consolidation due to constant rate of loading, which can be used for estimating the degree of consolidation and consolidation settlement more realistically. In addition, Terezaghi's empirical method to account for construction time in the case of constant rate of loading is revisited.

1.2 OBJECTIVE OF STUDY

The theme of the study is towards understanding of consolidation behaviour of thin clay laminae. These stiff thin clay laminae are consolidated to achieve final settlement either by Biot theory or by Terzaghi theory. When a soil layer is subjected to a compressive stress, such as during the construction of a structure, it will exhibit a certain amount of compression. The compression of soil with time depends upon number of reasons. The expulsion of pore fluids and rearrangement of soil structure are the main reasons behind.

According to Terzaghi (1943), "a decrease in water content of a saturated soil without replacement of the water by air is called consolidation." When clayey soils, which have a low coefficient of permeability, are subjected to a compressive stress due to construction, the pore water pressure will immediately increase; however, because of the low permeability of the soil, there will be a time lag between the application of load and the expulsion of the pore water and thus, the settlement. This phenomenon is called consolidation which is the main theme of present study.As additional long-term settlement problems are to be anticipated due to the potential secondary compression associated with this relatively less permeability of clay lamenae of matter.

However, despite increased interest on the behaviour of soil, little is yet known about the magnitude and characteristics of secondary compression of these deposits. Evaluation of clay layer secondary compression of marshy land is required for estimation of expected total settlement of permanent structure. Moreover, preloading technique through surcharge has been employed with some success as a mean of insitu improvement of engineering properties of clay layer. However, for this technique to be effective the compressibility of the clay layer complex needs to be thoroughly investigated.

In saturated clay, during undrained conditions the applied load at time $t = 0$ is resisted completely by the pore water. If a drainage conditions exists, then initial pore water pressure dissipates with time leaving the soil, again in a fully saturated state, the consequent reduction in the volume of the soil is approximately equal to the volume of water entering to the free drainage boundaries. The continuous increase in the effective stress at any given time equals to the decrease in the excess pore pressure.

The assumption, along with the continuity conditions of water movement, enables the creation of the necessary equation based on Fourier series that express the development of the consolidation process.

The settlement due to consolidation is the major controlling factor in the design of footings and foundations, some errors had been encountered by assuming consolidation only occurs in vertical direction. Any immediate settlement that occurs on application of the load is estimated using elastic theory.

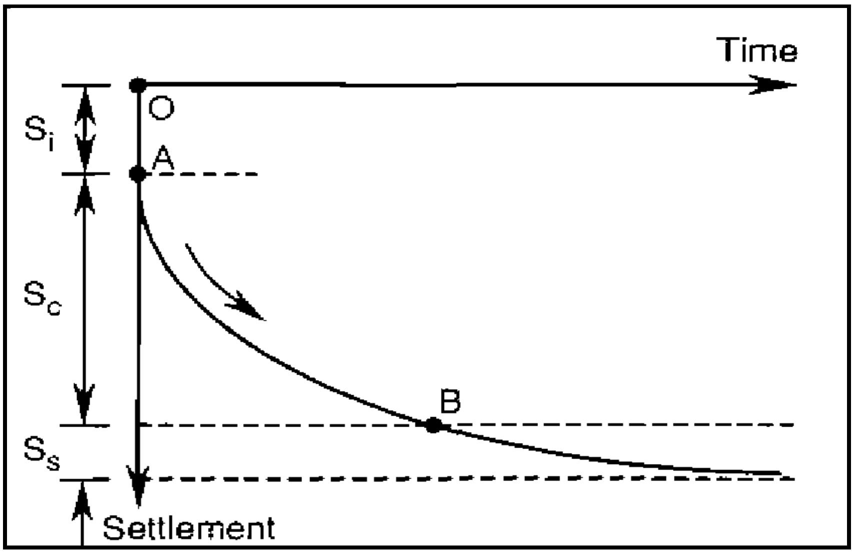

Fig 1.1: Time rate settlement curve for elastic loading

(Courtesy: Soil Mechanics basic concepts, Aysen A 2002 pp 214)

Where,

S_i = immediate settlement

S_c = consolidation settlement

S_g = secondary settlement

Above figure represents typical time settlement relationship for an element of saturated clay during a vertical load increment. The settlement or the vertical compressive deformation of the element is divided in to three segments S_i, S_c, S_g.

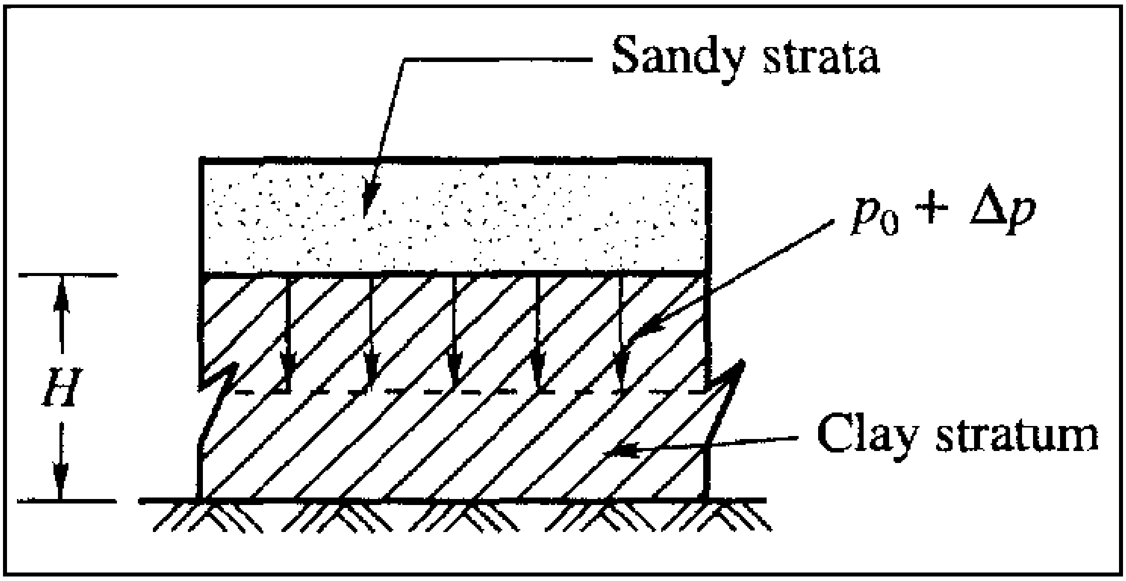

Fig 1.2: Effective stress applied on thin clay laminae

(Courtesy: Principles of Soil Mechanics, Murthy V N S 2000 pp 222)

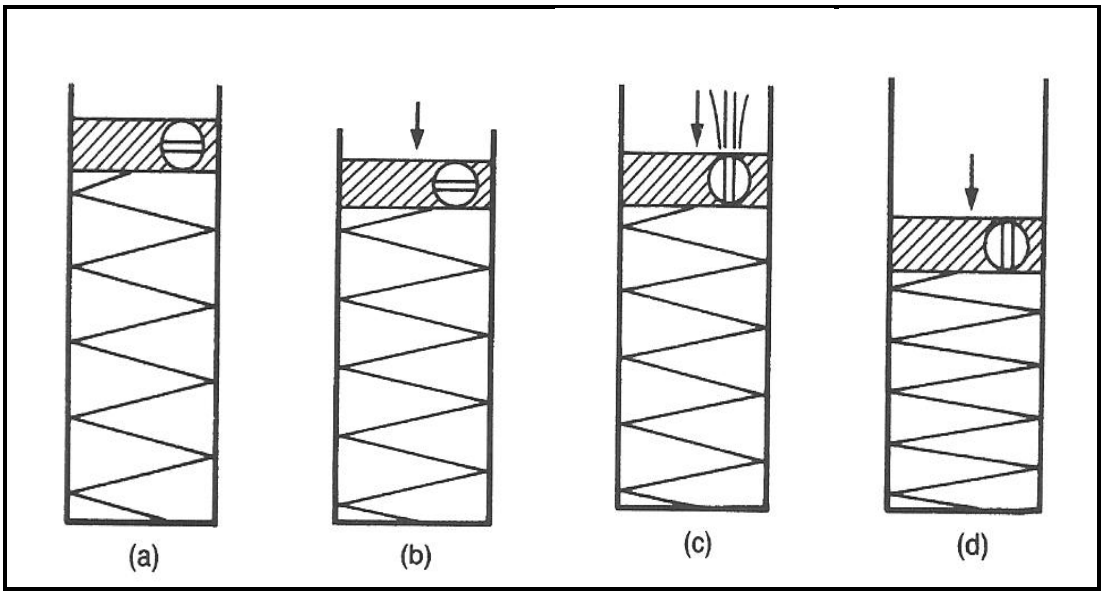

Fig 1.3: Mechanism of Consolidation

(Courtesy: http://soilworks.com/consol_1/introduction)

1.3 MAJOR SCIENTISTS AND THEIR CONTRIBUTION

Table 1.1: Major Scientists & their Contribution

Scientists	**Contribution**
Terzaghi's Analysis, 1925	1-D Consolidation
Biot Theory, 1940	3-D Consolidation with restructuring of soil solids
Ji et al., 1948	Obtain quasi-dynamic solution for symmetric consolidation of isotropic soil.
Schiffman, 1958	1-D consolidation theory under time dependant loading where the permeability and coefficient of consolidation vary with time.
Geng et al., 1962	Use of Laplace transforms to obtain solution for non linear 1-D consolidation.
Davis & Raymond, 1965	Relation of permeability of soil with coefficient of consolidation.
Gibson & Hussey, 1967	Analysis of 'rate of consolidation' depending upon thickness of soil strata.
Mikasa et al., 1968	1-D Consolidation theory in terms of compressive strains instead of excess pore pressure..

Booker and Small, 1976	Finite layer analysis technique for 2-D and 3-D consolidation of multilayered soils.
Oslon.et al., 1977	Incorporate Ramp loading as mode of load application in 1-D consolidation.
Cai et al., 1981	Semi analytical 1-D consolidation solution with variation of cyclically loaded soil compressibility.
Senjuntichai and Rajapakse, 1989	Solution for consolidation of multilayered soils by an exact stiffness method in the cylindrical coordinate system.
Sridharan et al., 1995	Analysis of Time factor for secondary consolidation of extremely clayey soil.
Wang and Fang, 2001	Analysis of Biot consolidation problem for multilayerd porous media by a state space method in cylindrical coordinate system.
Conte et al., 2006	Analysis of coupled consolidation of unsaturated soil under plain strain loading.
Rani et al., 2011	Consolidation of mechanically isotropic but hydraulically anisotropic clay layer incorporating compression of pore fluid and solid constituents.
Vinod et al., 2010	Cross section range of fast loading for radial consolidation.
Tewatia et al., 2012	Quick and fast loading methodology to guage/measure rate of consolidation of primary as well as secondary processes.

Proposed study (2013) – general solution of consolidation using Laurent series of thin clay laminae, determination of degree of consolidation with new sets of computations of varying domain also incorporated for multi layered soils.

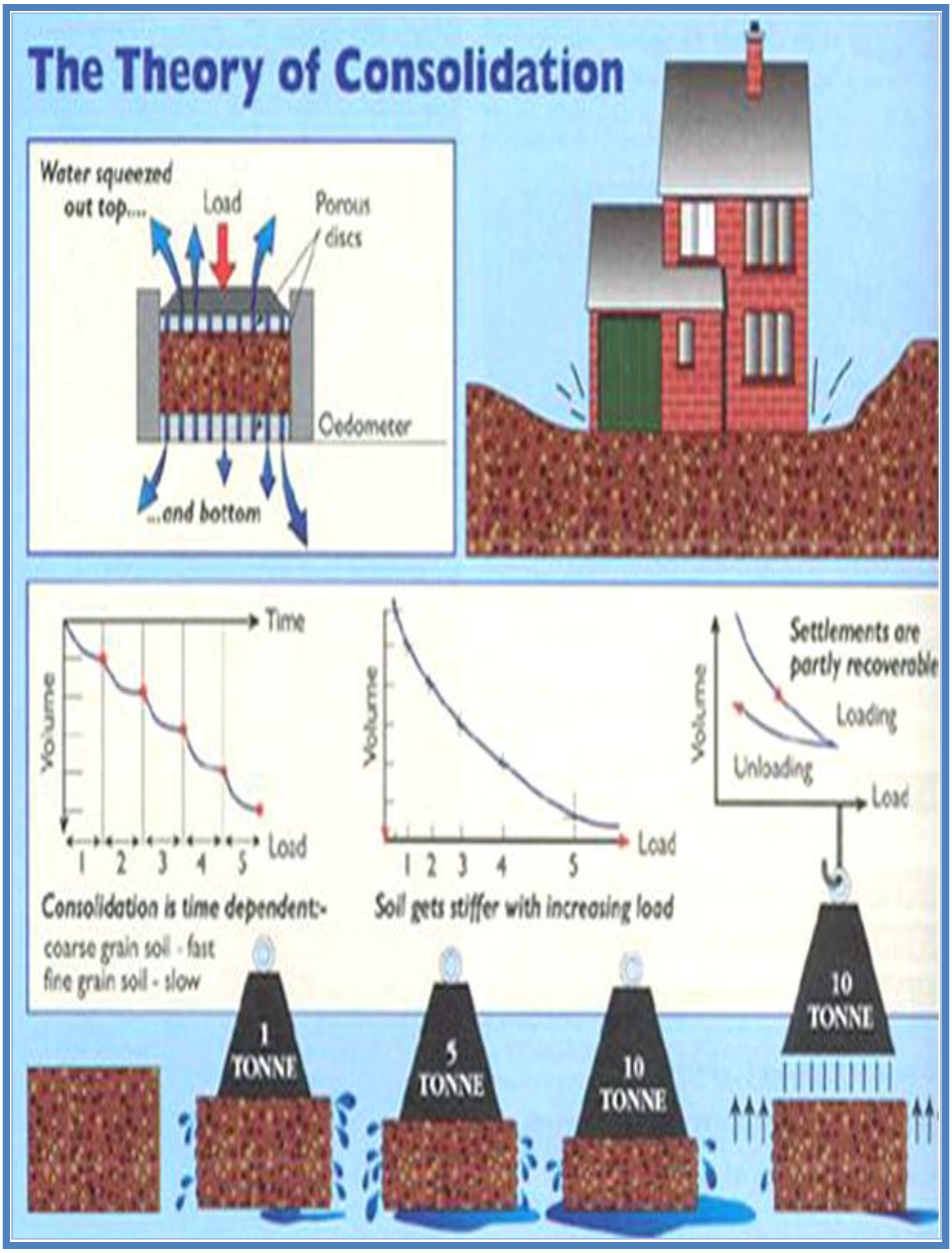

Fig 1.4: Consolidation with Terzaghi principle

(Courtesy: http://soilworks.com/consol_2/introduction)

1.4 PHYSICAL BACKGROUND: CORRELATION OF FOURIER (TERZAGHI) SERIES WITH LAURENT SERIES

(Courtesy: www.math.lsa.umich.edu/rauch/555/laurentfourier.pdf)

As it is clear from lots of mathematical researches that Fourier series can easily be converted in to Laurent series by multiplication of the constant$e^{i\theta}$. A tough factor to know or encourage is that the inconvertible fact discretional periodic functions have Fourier series representations. Within the following study it is shown that for periodic functions that are analytic, the illustrations follows from basic facts regarding Laurent series.

(a) Fourier series of periodic functions that are to be analytic.

A operate f (z) is amountic with period 2π if whenever z belongs to its domain therefore do the points z + 2πn with n ∈ Z and one has for all n, $\mathbf{f}(z + 2\pi n) = \mathbf{f}(z)$. We have an interest in functions that are analytic and outlined on an open set of C containing the real axis. The complement of the domain of f is then closed and disjoint from the compact interval [0, 2π] within the real axis. Opt for a > zero so these 2 closed sets are at distance ≥ a from one another. It follows that the domain of f contains a full strip $\{z: |\mathrm{Im}\, z| < a\}$, $a > 0$

Examples of periodic analytic functions: The elementary functions and e±inz are the building blocks. Any linear combination of the higher than non linear functions too, as an example f (z) = $\frac{1}{(sin\, z)^2}$ will be analytic in any strip on which $sin\, z \neq \pm$ i. Whole of the function h = $\sum_0^\infty a_n . z^n$ yields the complete example h (e^{iz})=$\sum_0^\infty a_n . e^{inz}$. These examples are often changed to yield the overall case as follows. Take into account the mapping w = e^{iz}.It maps out the above strip in the complex plane z to the annulus $\{w :\ e^{-a} < |w| < e^{a}\}$ in the w plane.

It maps the important axis within the z plane infinitely usually around the unit circle within the w plane, the preimages of a degree w = $e^{i\theta}$ are the points z = θ + 2πn with n ∈ Z. Since the by-product $\frac{dw}{dz}$ is obscurity zero it follows that the mapping is domestically invertible with analytic inverse. The local inverses are branches of the perform z = $\frac{(ln\, w)}{i}$.

Theorem: The correspondence f (z) = g (e^{iz}) establishes a 1 to 1 linking between the 2π periodic analytic functions f (z) in the strip itself and also the analytic functions g(w) on the annulus.

Proof: Each such term 'g' yields associate in nursing analytic periodic function 'f ' on the strip and that distinct functions g yield distinct f is obvious. What has to be shown is that each periodic analytic perform on the strip has such a illustration. Suppose that f is analytic and periodic within the strip. For every purpose w within the annulus, the preimages z underneath the map consist the strip and take issue by number multiple of 2π. It follows that as per formula g on the annulus is well outlined by the formula g (w) = f (z), since it does not matters that z one takes. For any w opt for a preimage z. The mathematical function theorem implies that w contains a native inverse z = f (w) analytic on the vicinity of w and satisfying f (w) = z. This shows that g is analytic at each w in order that g provides the required illustration of f.

Theorem: If f (z) could be a 2π periodic analytic perform within the strip then f contains a Fourier series illustration $f(z) = \sum_{n=-\infty}^{n=+\infty} c_n . e^{inz}$, with coefficients given by the formulae

$$c_n = \frac{1}{2\pi}\int_0^{2\pi} f(\theta). e^{-in\theta}\, \mathrm{d}\theta.$$

Proof: Choose g in order that on top of equation holds. Then use the Laurent enlargement of gg (w) = $\sum_{-\infty}^{\infty} c_n . w^n$, where $c_n = \frac{1}{2\pi i} . \oint \frac{g(w)}{w^{n+1}} . dw$ at $|w| = 1$.

Since f (z) = g (e^{iz}) , so we have f (z) = $\sum_{-\infty}^{+\infty} c_n (e^{iz})^n$. Parameterizing the curve |w| = 1 by

w = $e^{i\theta}$ with 0 ≤ θ ≤ 2π, one has dw = i w dθ and therefore the formula for c_n becomes

$$c_n = \frac{1}{2\pi i} . \oint \frac{g(e^{i\theta})}{w^{n+1}} . i\, w\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{f\,(\theta)}{w^n}\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} f\,(\theta) e^{-in\theta} d\theta.$$

(b) Fourier series of smooth periodic functions

To derive the Fourier illustration of smooth periodic and L^2 periodic functions future result suffices. The key step uses the Fourier enlargement of Associate in Nursing approximating analytic periodic perform.

Theorem: If f (θ) will be a smooth 2π periodic perform whose all Fourier coefficients vanishes, then we will take f = 0.

Proof: Choose normalizing coefficient $C_\in$ so that $C_\in$. $e^{-x^2/\epsilon^2} \to \delta$ as $\in \to 0$.Define $f_\in(z) = C_\in \int_{-\infty}^{+\infty} e^{-(z-\theta)^2/\epsilon^2} f(\theta)\, d\theta.$

From above it is clear that f is entire analytic in z and periodic in the domain of 2π.The Fourier coefficients of $f_\in$ vanish since,

$$\int_0^{2\pi} e^{-in\theta}. f_\in(\theta) d\theta = C_\in \int_0^{2\pi} e^{-in\emptyset}. e^{-(\emptyset-\theta)^2/\epsilon^2} f(\theta) d\theta\, d\emptyset.$$

Convert variable from θ to $\xi = \emptyset - \theta$ to indicate that the d∅ integral is equal to, $\int_0^{2\pi} e^{-in\emptyset}. e^{-\frac{\xi^2}{\epsilon^2}}. f(\xi + \emptyset) d\emptyset\, d\xi$. On converting variable $\eta = \xi + \emptyset$ shows that the d∅ integral is equal to,$\int_0^{2\pi} e^{-i\eta\emptyset} f(\xi + \emptyset) d\emptyset = \int_\xi^{\xi+2\pi} e^{-i\eta(\eta-\xi)} f(\eta) d\eta = e^{in\xi} f(\xi) d\xi = 0$,since the Fourier coefficients of f gets ended. The Laurent growth of analytic periodic functions then implies that $f_\xi = 0$.On the opposite hand, as $\in \to 0$, the restriction of $f_\in$ to the important axis converges uniformly to f proving that f = 0.

Conversion of Fourier series in to Laurent series

Suppose $f(z) = \sum_{k=-\infty}^{+\infty} c_k z^k$ is a Laurent series convergent on an annulus {z: 0 < $|z|$ < R}. If R > 1, the series converges on S^1, so a substitution z = $e^{i\theta}$gives Fourier series

u (ɵ) = f ($e^{i\theta}$) = $\sum_{k=-\infty}^{+\infty} c_k e^{ik\theta}$

Using Euler's formula, we may rewrite $c_{-k}e^{-ki\theta} + c_k e^{ki\theta}$as a linear combination of $cos(k$ɵ) and $sin(k$ɵ).For conversion purpose we may write

u (ɵ) = $\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k\, cos(k\theta) + b_k\, sin(k\theta)$. For real valued u, complexophobes use this form to avoid number together.Thus figures shown below represent conversion of basic Fourier series into Laurant series only due to incorporation of factor $e^{i\theta}$.

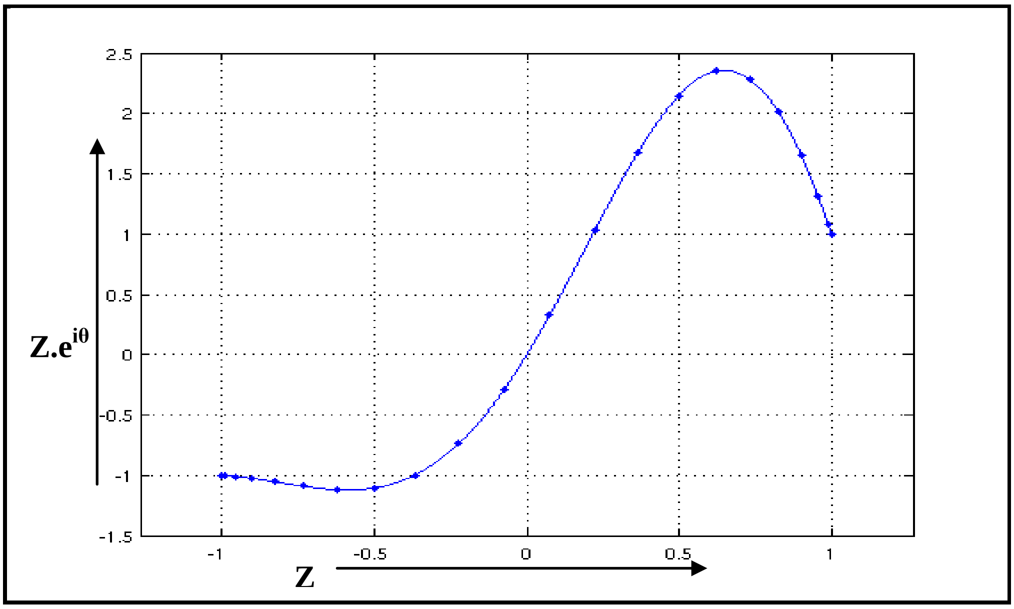

Fig 1.5: Variation of function 'z' with space and time on conversion from Fourier series to Laurent series.

(Courtesy: http://wikimedia.org/weikipedia/Fourier & Laurent series)

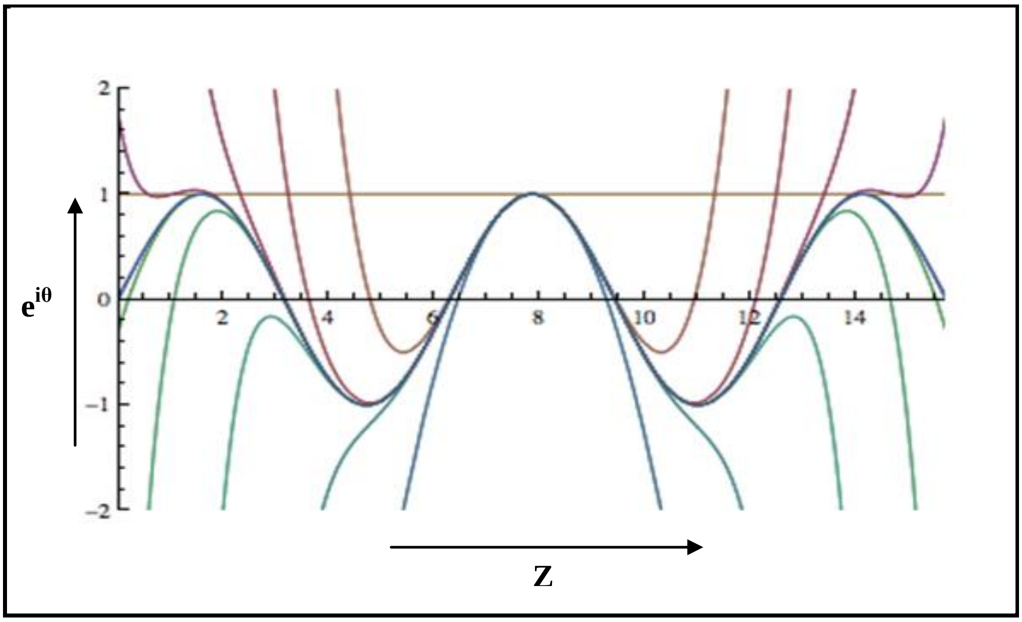

Fig 1.6: Variation of function 'z' with space and time on conversion from Fourier series to Laurent series in different domains.

(Courtesy: http://borborygmus.org/weikipedia/Fourier & Laurent series)

1.5 ORGANISATION OF REPORT

Chapter 1 It provides the introduction to the consolidation process, how it occurs and factors on which it depends. Chapter basically deals with the mechanism of the consolidation process and settlement that took place as a result of the same. It also gives the information about the idea that is to be incorporated to convert the Fourier series in to Laurent series and use it in terms of soil and its behaviour during load application.

Chapter 2 Describes about the literature reviews of various methods and techniques that are carried out by renowned researchers in past few decades for determination of rate of consolidation. Various researches had been carried out in determination of 3-dimensional consolidation. Numerical techniques like FEM, BEM and other matrix solutions used for determination of rate of settlement till now is well explained in the chapter.

Chapter 3 It deals about the numerical analyses that are to be used for determination of rate of consolidation. Precise and clear vision had been given about Laurent series and transformations and its applicability along with correlations.

Chapter 4 Discussed about the experiments that has been conducted on different sets of sample, along with different engineering properties of soil that are determined in the laboratory. All the experiments carried out here is according to I.S. Codes.

Chapter 5 In this chapter, result and validation were explained that correlates numerical Laurent transformations with experimental data obtained in laboratory, various numerical relations had been derived for consolidation process and is proved to be true when compared analytically as well as graphically.

Chapter 6 This chapter includes the conclusion part of the project work which is just the summary of different theories evolved in the process of consolidation along with limitations that arouses in numerical as well as experimental technique. Future scope of the work is also provided at last.

Chapter 7 Here an account of several research publications has been made, which we had referred during the study.

CHAPTER – 2

LITERATURE REVIEW

2.1 CONSOLIDATION: NATURAL PHENOMENON

Consolidation is likely to be linked to the changes in effective stress, which result from the changes in pore-water pressure as seepage flow progresses toward the drainage boundaries. Upon application of an external load, there is an initial increase in the pore-water pressure throughout the sample known as the initial excess pore water distribution. According to Darcy's law, the excess pore water pressures i.e., pressures in excess of hydrostatic are the driving force of seepage flow.

Flow takes place due to the hydraulic gradient generated by the initial excess pore pressure distribution. At any stage of the consolidation process, the pore-water pressures will vary within the soil layer. The distribution of the excess pore-water pressure at any given time after loading can be represented by an isochrones **[Powrie (1997**)]. Thus, the process of consolidation can be expressed as a series of the isochrones that graphically represent the relationship between time and the degree of consolidation over the depth of the soil stratum. Consolidation can be further characterized by the average degree of consolidation, which represents the consolidation of the stratum as a whole and eliminates the variable of depth**. [Taylor (1962)]**. Terzaghi's one-dimensional consolidation theory is commonly adopted to describe the dissipation of the excess pore-water pressure within a consolidating soil over time **[Terzaghi (1925**)].

In fact, Terzaghi's theory is often used in two and three-dimensional problems. Here, one-dimensional consolidation settlements are simply modified by a correction factor proposed by **[Skempton and Bjerrum (1957)]** to account for two- and three-dimensional effects. The behaviour of a consolidating soil subjected to the uniform initial excess pore-water pressure is commonly described in geotechnical textbooks in terms of both degree of consolidation isochrones and average degree of consolidation curves were given by **[Terzaghi (1943); Taylor (1962); Holtz and Kovacs (1981); Berry and Reid (1988); Powrie (1997); Atkinson (2007);**

Lancellotta (2009)]. The average degree of consolidation behaviour of other initial excess pore-water pressure distributions has also been analyzed in terms of one-dimensional consolidation.

2.2 GENERAL THEORIES

Since the inception of Terzaghi's 1-D consolidation theory, several solutions have been developed for dealing with issues relating to the consolidation resulting from time-dependent construction loads. This has led time-dependent construction loads to be approximated as constant rate loading scenarios **[Terzaghi (1943); Schiffman (1958); Olson (1998); Zhu &Yin (1998); Conte & Troncone (2006); Hsu & Lu (2006)]**. They range from semi-empirical approximations to more sophisticated theoretical analyses. The simplest of these approaches is that proposed by [**Terzaghi (1943**)] where there is the consolidation settlement at time t ($t < t_0$) is computed assuming that the pressure at that time is applied instantaneously at t/2. Due to its simplicity, this method is still discussed in different textbooks, with suggestions to implement this as a graphical procedure that accounts for the construction time [**Craig (2004**)].

Later, more rational and analytical approaches were proposed by several others which are summarised here briefly [**Olson (1977**)] extended Terzaghi's one-dimensional consolidation theory to incorporate ramp loading, covering consolidation due to vertical flow, radial flow and combined vertical and radial flow. He discredited the ramp loading into very small incremental loads, and applied Terzaghi's one-dimensional consolidation theory and principle of superposition, to develop a mathematical expression for the excess pore water pressure. This was used in developing an expression for the degree of consolidation in terms of time factor. The solutions were also presented graphically in the form of U-T charts. In Terzaghi's one-dimensional consolidation theory, as well as [**Olson's (1977**)] extension, it is assumed that cv remains the same during consolidation.

[**Schiffman (1958**)] studied one-dimensional consolidation under time dependent loading where the permeability and coefficient of consolidation vary with time.[**Zhu and Yin (1998**)] developed solutions for one-dimensional consolidation under depth dependent ramp loading, where the applied pressure varies with time and depth.[**Conte and Troncone (2006**)] developed a solution for consolidation due to more general time dependent loading. [**Mikasa (1968**)] developed the one-dimensional consolidation theory in terms of compressive strains instead of excess pore pressures, which was later extended to multi-layered clays by [**Kim and Mission (2011**); **Conte (2006**)] proposed a method to analyse coupled consolidation of unsaturated soils under plane strain and axi-symmetric loading.

[**Rani (2011**)] studied the consolidation of a mechanically isotropic but hydraulically anisotropic clay layer subjected to axi-symmetric surface loads, allowing for compressibility of the pore fluid and solid constituents. Soil consolidation is often caused by external loadings, such as in building or embankment construction on clayey soil. Consolidation theory was originally developed by Terzaghi for the one dimensional case, and therein, exclusively considered vertical stress and strain, and neglected horizontal effects. Biot later extended Terzaghi's theory to two- and three-dimensional saturated soils.

Biot's reformalization of Terzaghi's theory is also contemporarily referred to as the true two- or three-dimensional consolidation theory, as it permits the total stress to be varied as a function of time during the consolidation process. Both theories assume that loading is instantaneously applied and maintained constant as a function of time; however, realistic loadings in construction are usually applied gradually as a function of time. In many cases, the loading process may proceed over a long period of time, such that a significant part of the consolidation may occur during the loading process. Moreover, during the construction of some special structures, such as silos or fluid tanks, the soils beneath will be subjected to cycles of loading and unloading that periodically repeat over time.

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For example, cyclical loading was evaluated and the corresponding solutions therein were developed. In particular, [**Geng (1962**)] used the Laplace transform to obtain a general solution to the non-linear one-dimensional consolidation problem for soils that have undergone complicated cyclical loading.[**Conte (1966**)] used the Fourier transform to develop an analytical solution to the one-dimensional consolidation of saturated soil layers subjected to a general time-dependent loading. [**Cai (1981**)] derived a semi-analytical one-dimensional consolidation solution by considering the variation of cyclically loaded soil compressibility.

Process over time, wherein these loadings can significantly change the total stress as the soil consolidation develops. As a result, the impact of the external loading type should be considered in the analysis. Several solutions have been developed for classical one-dimensional consolidation problems consisting of time-dependent loading conditions in the past few decades. [**Schiffman (1958**)] was one of the first researchers to obtain a general solution of consolidation with loadings that linearly increased with time, which has subsequently been modified by later work. For example, cyclical loading was evaluated and the corresponding solutions therein were developed.

In Several investigators have successfully obtained analytical solutions to Biot's equations; however, due to the complexity of the coupled governing differential equations, the solutions to consolidation problems are generally limited to those with simple geometries and specific loading conditions. Several investigators have successfully obtained analytical solutions to Biot's equations; however, due to the complexity of the coupled governing differential equations, the solutions to consolidation problems are generally limited to those with simple geometries and specific loading conditions. Earlier work on three-dimensional consolidation problems has typically adopted a purely isotropic elastic model for the soil skeleton; however, the soil is often transversely isotropic due to its original deposition in horizontal bedding.

Furthermore, imposed strains after deposition can induce additional anisotropy, which in turn leads to a preferred orientation of the plate-shaped clay particles, and has been verified by laboratory tests that show that the stiffness and permeability of soils are directionally dependent. Thus, a more realistic solution to the consolidation problem should accommodate soil anisotropy. Existing solutions to consolidation problems all assume that particles and pore water are incompressible, which may not always be appropriate. In general, it is necessary to relax this assumption, and extend consolidation problem analysis to compressible materials. In this vein, [**Ji (1948**)] obtained a quasi-dynamic solution for the axi symmetric consolidation of a columnar cross-isotropic soil. More recently, [**Chen (1954**)] derived solutions for the axi-symmetric consolidation of transversely isotropic saturated soils using Laplace and Hankel transform techniques, but under instantaneously applied loading that was held constant with time.

Until now, investigations into the consolidation behaviour of semi-infinite transversely isotropic soils are sparse in literature, especially in regard to soils subjected to complex time-dependent loadings. Therein, soil particles and water pore compressibility and loading histories that more satisfactorily represent consolidation problems were analyzed. [**Chen (1954**)] originally derived these formulations for constantly loaded scenarios. Several numerical examples were investigated to verify the proposed approach, and therein, identified the influence of material anisotropy on pore pressure dissipation and soil settlement. Since the development of the three-dimensional consolidation theory by Biot in 1941, it has been used by researchers to solve many consolidation problems of soils. For example, [**McNamee and Gibson (1960**)] obtained the analytical solutions for the plane strain and axi-symmetrical consolidation problems with a semi-infinite body.

[**Schiffman and Funguroli (1968**)] studied the consolidation of a half-space medium on which a uniform tangential load was applied in the cylindrical coordinate system.[**Gibson and Booker (1974**)] proposed the analytical solutions for the consolidation of a finite layer subjected to surface loading.[**Yue and Selvadurai (1980**)] analyzed the interaction between a circular, flat rigid indenter and a poro-elastic half-space saturated with a compressible fluid. For multi-layered soils, [**Vardoulakis and Harnpattanapanich (1986**)] adopted a numerical method using displacement functions and integral transforms to analyze two-dimensional and three-dimensional consolidation problems in the Cartesian coordinate system.[**Booker and Small (1976**)] developed a finite layer analysis technique to deal with two-dimensional and three-dimensional consolidation problems of multilayered soils.

[**Senjuntichai and Rajapakse (1989**)] solved the consolidation problems of multi-layered soils by an exact stiffness method in the cylindrical coordinate system.[**Pan(1990**)] employed vector functions and a propagator matrix method in the cylindrical and Cartesian coordinate systems to obtain Green's functions in a multi-layered, isotropic, and poro-elastic half space.[**Wang and Fang (2001**)] analyzed the Biot consolidation problem for multi-layered porous media by a state space method in the cylindrical coordinate system. More recently [**Ai and Han (2003**)] used a state space method to obtain the solution for the plane strain consolidation of multi-layered soils. Numerical methods, such as the finite element method and the boundary element method have also been used to solve these complicated consolidation problems. Since Terzaghi printed his consolidation theory and therefore the principle of effective stress, analysis work on consolidation issues has greatly inflated.

The conventional consolidation theories typically neglected the non-linearity of soil for sensible functions. Studies of non linear consolidation behaviour of sentimental soil started from concerning forty years past. Many efforts are created to get analytical solutions for various forms of one-dimensional consolidation theories**.**

[Davis and Raymond (1965)] derived associate analytical resolution for the opposite constant loading case supported the assumptions that the decrease in porousness is proportional to the decrease in squeezability throughout the consolidation of a soil and therefore the distribution of initial effective pressures is constant with depth. With a similar assumption concerning the squeezability and porousness of a soil, **[Xie (1965)]** developed associate analytical resolution to the one dimensional consolidation drawback for time-dependent loading. the answer given by Davis and Raymond was considered a special case of it. **[Poskitt (1971)]** studied a lot of general one dimensional non linear consolidation drawback by employing a perturbation methodology, however no express resolution was given. For the linear consolidation of superimposed soils, some analytical solutions are reported. However, there appears to be no analytical resolution for nonlinear consolidation taking the superimposed characteristics of soil into thought. This could be primarily owing to mathematical problem. During this study, associate analytical resolution comes for 1-Dimensional non linear consolidation of double-layered soil considering time-dependant loading, supported a similar assumption projected by Davis and Raymond apart from loading condition, and everyone the analytical solutions up to now developed square measure summarized within the tables. The nonlinear consolidation behaviour of double-layered soil is additionally mentioned.

The modelling and prediction of the mechanical behaviour of structures like building foundations, embankments, oil- cans, silos and ground anchors resting on a saturated layered soil system area unit common and essential issues in geotechnical engineering. Saturated soil consists of 2 phases, namely, a solid part (the soil skeleton) and a liquid part. These 2 phases act once the saturated soil is subjected to associate in nursing external load.

[Biot (1941)] developed a theory that may justify the complicated interaction between the solid and also the fluid, i.e., the supposed Biot's consolidation theory. This theory will accurately describe the method of soil consolidation during a three-dimensional condition. Within the past few decades, several researchers have advanced the data and theory of soil consolidation. For most mechanisms planned to make a case for secondary effects, one would expect a lot of noticeable secondary result within the laboratory than within the field. Laboratory values of c_v (and k) area unit seemingly to be too low as a result of retarding secondary effects area unit seemingly to be rather more vital within the laboratory than within the field because of the upper strain rate within the laboratory **[Taylor (1942); Barden (1965); Lo et**

al. (1976); Poskit (1967)]. The time needed to finish the check victimisation the speedy consolidation methodology [**Sridhar an (1999)]** can be as low as 4-5 h compared with 1 or 2 weeks within the case of the traditional consolidation check on extremely clayey soils.

Rectangular conic section methodology needs knowledge of concerning 70% U for decisive c_v, δ_{100} etc. Where, c_v and δ_{100} area unit constant of vertical consolidation and supreme primary settlement, severally. However, the c_v values area unit not up to true c_v due the consequences of secondary consolidation as secondary consolidation primarily starts at 60% U **[Sridhar an et al. (1995)]**. Also, it is not legendary to what extent c_v values area unit stricken by secondary consolidation. The trend or rate of settlement of pressure acting on clay will be determined at any time while not knowing the past history of pressure increment. It will be a supply of some helpful data like fast analysis of consolidation characteristics within the laboratory and field, time-compression knowledge of the current, past and future, kind and stage of consolidation, emptying conditions, time of load increment etc. **[Tewatia (2012)]**.

Using the speed of settlement idea **[Tewatia (1998); Tewatia and Satyendra N. Bose (2006)]**, a quickest fast loading methodology is projected to guage c_v minimizing the results of secondary consolidation that offers some estimate additionally that to what extent c_v is plagued by secondary consolidation. There square measure range of fast loading ways for vertical consolidation however few exist for radial consolidation **[Vinod et al. (2010); Tewatia et al. (2012a)]**. A fast loading methodology is projected for radial consolidation that's abundant quicker than the ways offered in literature with the higher than mentioned deserves additionally. The projected ways will be used even once the settlement or time of load increment is not glorious.

2.3 APPLICATION OF FOURIER SERIES

Rate of Consolidation shows the erratic behaviour of the Fourier series of a piecewise continuously differentiable periodic function at a discontinuity [**Carslaw (1930**)]. When initial excess pore pressure distributions contain discontinuities (i.e., are not zero at the stratum boundaries), the n^{th} partial sum of the Fourier series exhibits oscillations near the discontinuity. A discontinuity of the initial excess pore pressure distribution exists at the base of the soil stratum. Consequently, a discontinuity will exist wherever the initial excess pore pressure at a boundary is greater than zero, as is the case for singly drained soil strata where only one drainage boundary exists. Fourier Series is present in the form of oscillations at the discontinuity, where the series tends to overshoot/undershoot the positive corner by as much as 18% [**Arfken (1970**)]. The inclusion of more terms does not remove this oscillatory effect, but merely moves it closer to the point of discontinuity.

Numerical methods, such as the finite element method were also used to solve complex consolidation problems. In all these studies mentioned above, however, it was assumed that the pore fluid is incompressible, which is only an extreme case of the generalized model of compressible pore fluid. Actually, absolutely saturated soil does not exist. There is always pore-gas phase in the liquid phase, so the pore fluid of liquid phase should be considered as compressible. Some researchers demonstrated the influence of pore fluid compressibility on the consolidation. For example, **[Cheng and Liggett (1984)]** concluded that the compressibility of pore fluid has a great influence on both the consolidation process and the pore pressure distribution. **[Yue (1994)]** demonstrated that the presence of a compressible pore fluid reduces the generation of excess pore pressure in a poroelastic sea bed layer.

[Senjuntichai and Rajapakse (1995) and **Chen (2004)**] also demonstrated the influence of fluid compressibility on the axisymmetric and non-axisymmetric consolidation of a multi-layered poro elastic medium. Biot's three-dimensional consolidation problem of a multi-layered soil with compressible constituents in the Cartesian coordinate system is investigated by using the displacement function method and the transfer matrix method. **[Conte (2006)]** given a technique for the analysis of coupled consolidation in unsaturated soils because of loading below conditions of plane strain in addition as axial symmetry. The strategy relies on the re-work of the governing differential equations by the Fourier transform, once the soil system is distorted below plane strain conditions, or Hankel rework for issues exhibiting axial symmetry. The impact of such transformations is to modify significantly the answer from a machine purpose of read.

CHAPTER – 3

NUMERICAL ANALYSIS

3.1. FOURIER SERIES AND TRANSFORMATIONS

These are the basic tool for solving ordinary differential equations (ODE) & partial differential equations (PDE) with periodic boundary end condition. Chronologically the history of Fourier series is described as follows:

- 1700 Sauveur — experiments with harmonics
- 1740 Daniel Bernoulli (St. Petersburg) — superposition of harmonics
- 1747 contention between Euler and d'Alembert
- 1807 Fourier — formula for coefficients
- 1829 Dirichlet — initial convergence proof
- 1965 Carleson — nearly everyplace pointwise convergence for square summable functions

Fourier series is very well defined as an expansion or dilation of a function or representation of a given oscillatory function in a form of discontinuous series.

F (z) = $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n . \cos nx + \sum_{n=1}^{\infty} b_n . \sin nx$ (4)

Where coefficients a_0 , a_n , b_n are related to the periodic function f(x) by definite integrals

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x). \cos nx. dx \quad (5)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x). \sin nx. dx \quad (6)$$

Where n = 0, 1, 2

Equation 1 can be valid only when conditions imposed on f(x) such that f(x) had finite number of finite discontinuities & finite number of extreme values maxima & minima in the interval (0,2π) i.e.

$$\sum_{n=1}^{\infty} \frac{1}{n} \cos nx \,, x \in (0,2\pi) \qquad (7)$$

Since this series is only convergent at defined conditions (and diverges at x=0) we take

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n} = \lim_{r \to 1} \sum_{n=1}^{\infty} \frac{r^n . \cos nx}{n} \,, \qquad (8)$$

Absolutely convergent for $|r| < 1$.The birth of Fourier series can be traced back to the solutions of wave equation in the work of Bernoulli and the heat equation in the work of Fourier. Consider an elastic string of finite length ‘l’ fixed at the end points x = 0 and x = l. At time, say t = 0, it is distorted from the equilibrium position and allowed to vibrate. The problem is to find the vibrations of the string at any point x and any time t > 0. The vibration of the elastic string is governed by the linear sets of partial differential equation as shown here.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad (9)$$

$$u(x,0)=f(x) \qquad (10)$$

$$\frac{\partial}{\partial t} u(x,0) = g(x) \qquad (11)$$

Where c^2 = constant, f(x) = initial position, g(x) = initial velocity.

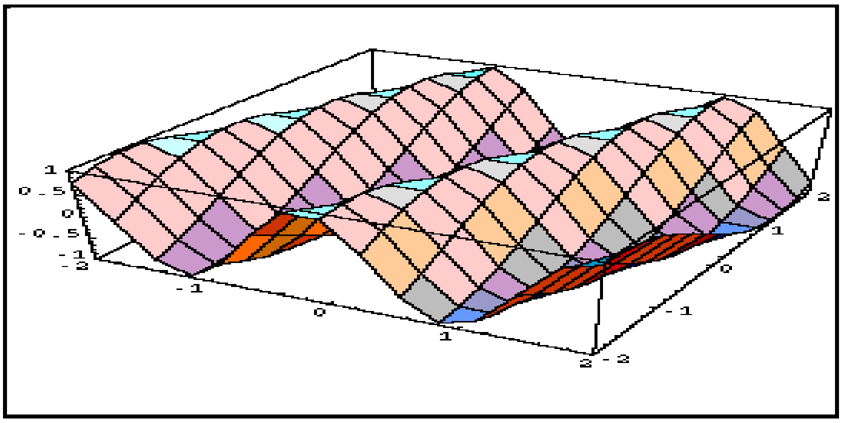

Fig 3.1: Progression of Vibration string in Fourier series

(Courtesy http://jowett.home.cern.ch/jowett/ComputingNotes/wave.gif)

3.2. THEORY OF CONSOLIDATION USING FOURIER SERIES

By means of Fourier series solution of the differential equation of consolidation is obtained: The solution must satisfy the following hydraulic boundary equations:

At t=0, at any distance z (constant)

At t=∞, at any distance z and

At t=t, at z=0, and at z=H (12)

If ū is assumed to be a product of some function of z and t, it may be represented by the following expression:

$$\bar{u} = f_1(z).f_2(t) \tag{13}$$

Equation of consolidation is given by:

$$S\frac{\partial \bar{u}}{\partial t} = C_v \tag{14}$$

And above equation can be written as

$$f_1(z).\frac{\partial u}{\partial t}[f_2(t)] = c_v f_2(t).\frac{\partial^2}{\partial z^2}[f_1(z)] \tag{15}$$

or $$\frac{\frac{\partial^2}{\partial z^2}[f_1(z)]}{f_{1(z)}} = \frac{\frac{\partial}{\partial t}[f_2(t)]}{c_v f_2(t)} \tag{16}$$

In above equation, L.H.S does not contain t and hence it is taken as constant if t is considered variable. Similarly, the R.H.S is constant when z is considered variable. Hence each term must be equal to a constant ($-A^2$) and hence represented by the following relations:

$$\frac{\partial^2}{\partial z^2}[f_1(z)] = -A^2 f_1(z) \tag{17}$$

$$\frac{\partial^2}{\partial t^2}[f_2(t)] = -A^2.c_v.f_2(t) \tag{18}$$

Equation (17) and (18) can be satisfied respectively by the following:

$$f_1(z) = C_1 \cos Az + C_2 \sin Az \quad (19)$$

$$f_2(t) = C_3 e^{-A^2.c_v.t} \quad (20)$$

Where, $C_1 C_2$ C_3 = arbitrary constants, e = base of hyperbolic = 2.718

Substituting the above values, Eqn. (13) becomes:

$$\bar{u} = (C_4 \cos Az + C_5 \sin Az).e^{-A^2.c_v.t} \quad (21)$$

The solution of Eqn. (21) must satisfy the boundary conditions stated in Eqn. (12).

Thus at time t when z = 0, ū = 0, $C_4 = 0$.Hence Eqn. (21) reduces to:

$$\bar{u} = C_5(\sin Az).e^{-A^2.c_v.t} \quad (22)$$

Also, at time t when z = H, ū = 0 = $C_5(\sin AH).e^{-A^2.c_v.t}$ (23)

Equation (13) is satisfied when AH = nπ where n is any integer.

Hence ū = $C_5(\sin\frac{n\pi z}{H}).e^{-\frac{n^2.\pi^2}{H^2}.c_v.t}$ (24)

The above expression may be written in the following form:

$$\bar{u}=B_1(\sin\frac{\pi z}{H}).e^{-\frac{\pi^2}{H^2}.c_v.t}+B_2(\sin\frac{2\pi z}{H}).e^{-\frac{4\pi^2}{H^2}.c_v.t}+\ldots\ldots\ldots.+B_n\left(\sin\frac{n\pi z}{H}\right).e^{\frac{n^2.\pi^2}{H^2}.c_v.t}+\ldots\ldots$$

Or ū = $\sum_{n=1}^{n=\infty} B_n(\sin\frac{n\pi z}{H}).e^{\frac{n^2.\pi^2}{H^2}.c_v.t}$ (25)

When t=0, $e^{\frac{n^2.\pi^2}{H^2}.c_v.t} = 1$ and ū = $\bar{u}_0$

Thus, $\bar{u}_0 = \sum_{n=1}^{n=\infty} B_{n.} \sin(\frac{n\pi z}{H})$ (26)

We can determine B_n for the above Fourier series by the following mathematical relations:

$$\int_0^{\pi}(\sin mx \sin nx).dx = 0 \quad (27)$$

$$\int_0^{\pi}(\sin nx)^2.dx = \frac{\pi}{2} \quad (28)$$

Where, m≠n, $x \rightarrow \frac{\pi z}{H}$, $dx \rightarrow \frac{\pi}{H}.dz$, multiplying both sides of equation (26) by $sin\frac{n\pi z}{H}$ and integrating between the limits 0 to H:

$$\int_0^H \left(sin\frac{m\pi z}{H} sin\frac{n\pi z}{H}\right).dz = 0 \quad (29)$$

$$\int_0^H (sin\frac{n\pi z}{H})^2 .dz = \frac{H}{2} \quad (30)$$

$$\int_0^H \left(\bar{u}_0. sin\frac{n\pi z}{H}\right).dz = \sum_{m=1}^{m=\infty} B_m. \int_0^H \left(sin\frac{m\pi z}{H} sin\frac{n\pi z}{H}\right).dz + B_n \int_0^H (sin\frac{n\pi z}{H})^2 .dz \quad (31)$$

On multiplication by $sin\frac{n\pi z}{H}$ the R.H.S of Eqn. (26) splits in to two parts:

(i) – n-term which is of the form of equation (30).

(ii) – Series of all terms except n-term of the form (29), and vanishes.

Hence $\int_0^H \left(\bar{u}_0. sin\frac{n\pi z}{H}\right).dz = B_n.\frac{H}{2}$ (32)

or $B_n = \frac{2}{H}\int_0^H \left(\bar{u}_0. sin\frac{n\pi z}{H}\right) dz$ (33)

Substituting this in equation (25), we get

$$\bar{u} = \sum_{n=1}^{n=\infty} \left[\frac{2}{H}\int_0^H \left(\bar{u}_0. sin\frac{n\pi z}{H}\right).dz\right].\left(sin\frac{n\pi z}{H}\right).e^{-\frac{n^2.\pi^2}{H^2}.c_v.t} \quad (34)$$

or $\bar{u} = \sum_{n=1}^{n=\infty} \frac{2\Delta\sigma}{n\pi}(1 - cos\, n\pi).\left(sin\frac{n\pi z}{H}\right).e^{-\frac{n^2\pi^2}{H^2}.c_v.t}$ (35)

When n = even, (1-$cos\, n\pi$) = 0, n = odd

Substituting n = 2N+1, N = integer, above equation becomes:

$$\bar{u} = \frac{4}{n}.\Delta\sigma.\sum_{N=1}^{N=\infty}\frac{1}{2N+1}\left[sin\frac{(2N+1)\pi z}{H}\right].e^{-\frac{n^2.\pi^2}{H^2}.c_v.t} \quad (36)$$

Above equation represents variation of excess hydrostatic pressure ū with depth z at any time in terms of applied consolidating pressure $\Delta\sigma$.

Now the consolidating pressure, $\Delta\rho = m_v.\Delta\sigma'.dz$ (37)

Where $\Delta\sigma'$= effective pressure increment at time t, $\Delta\sigma' = \Delta\sigma - \bar{u}$

Thus $\Delta\rho = m_v.(\Delta\sigma - \bar{u}).dz$

Integrating between the limits 0 to H, the settlement ρ of the fully thickness of the clay at times t is given by: $\rho = m_v\left[\Delta\sigma.H - \int_0^H \bar{u}.dz\right]$

Substituting ū from equation (36) and integrating,

$$\rho = m_v.\Delta\sigma.H\left[1 - \frac{8}{\pi^2}.\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2}.e^{-\frac{(2N+1)^2.\pi^2}{H^2}.c_v.t}\right] \quad (38)$$

At t = ∞, when process of consolidation is complete, the ultimate or final settlement ρ_f is given by: $\rho_f = m_v.\Delta\sigma.H$ (39)

The ratio of ρ to ρ_f expressed as a percentage, is termed the "Degree of Consolidation" U; U (%) = $\frac{\rho}{\rho_f} \times 100$ (40)

$$\text{U (\%)} = \left[1 - \frac{8}{\pi^2}.\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2}.e^{-\frac{(2N+1)^2.\pi^2}{H^2}.c_v.t}\right] \times 100 \quad (41)$$

A dimensionless parameter is being introduced known as time factor T_v defined by following equation: $T_v = \frac{c_v.t}{d^2}$ (42)

Where, d=drainage path for present case of double drainage, d = H/2 Eqn. (40) may be written as:

$$\text{U(\%)}=\left[1 - \frac{8}{\pi^2}\sum_{N=0}^{N=\infty}\frac{1}{(2N+1)^2}.e^{-\frac{(2N+1)^2.\pi^2}{4}.T_v}\right] \times 100 \quad (43)$$

Or U (%) = f (T_v) (44)

3.3. LAURENT SERIES AND TRANSFORMATIONS

The theorem says that if we have a function f that is holomorphic on an annulus then it can be expressed as a Laurent series. A holomorphic function f may be expressed as a Taylor series i.e. if it is differentiable on a domain D and $z_0 \in D$ &

we can write it as

$$f(z)=\sum_{n=0}^{=} a_n.(z-z_0)^n \tag{45}$$

for appropriate constants a_n, and the above equation is valid for $|z - z0| < R$, for some $R > 0$.

The idea of Laurant series is to generalise to allow negative powers of $(z - z0)$. And hence turns out to be a very crude tool.

A Laurent series is a series that expand as $\sum_{n=-\infty}^{+\infty} a_n .(z-z_0)^n$ (46)

As (13) is a two times infinite sum, so we need to take care as to what it means. We define equation (13) to mean

$$\sum_{n=1}^{\infty} a_n.(z-z_0)^{-n} + \sum_{n=0}^{\infty} a_n .(z-z_0)^n = \sum^{-} + \sum^{+} \tag{47}$$

In the above equation both $\sum^{+}$ & $\sum^{-}$ are going to be converged. Now $\sum^{+}$ converges $|(z-z_0)|\langle R_2$ for some $R_2 \geq 0$,where R_2 = Radius of convergence of $\sum^{+}$ & $\sum^{-}$ has radius of convergence $R_1^{-1} \geq 0$ i.e. $\sum^{-}$ converges when $|(z-z_0)^{-1}|\langle R_1^{-1}$ or in other words $\sum^{-}$ converges when $|z-z_0|\rangle R_1$.

Combining these, we see that if $0 \leq R_1 \leq R_2 \leq \infty$ then, equation (46) converges in the annulus. Suppose that f is holomorphic in the annulus $\{z \in C \mid R1 < |z - z0| < R2\}$, where $0 \leq R1 < R2 \leq \infty$. Then we can write f as a Laurent series: for $R1 < |z - z0| < R2$.

We have $f(z) = \sum_{n=1}^{\infty} a_{-n} (z-z_0)^{-n} + \sum_{n=0}^{\infty} a_n (z-z_0)^n$ (48)

Equation (48) is known as Laurent series of f (z) about z_0 or the Laurent Expansion of f(z).

Where $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ is the principle part of Laurent series.

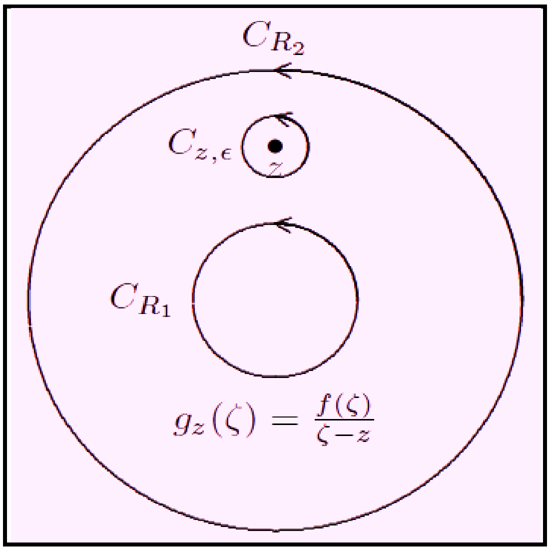

Fig 3.2: Domain of Laurent series

(Courtesy: Math 20101 complex analysis, Zhu University of Technology (pp- 57, 58))

3.4 THEORY OF CONSOLIDATION USING LAURENT SERIES

With the help of Laurent series, theory of consolidation & its solution can be given as

The solution must satisfy the following hydraulic boundary equations:

At t=0, at any distance z (constant)

At t=∞, at any distance z and

At t=t, at z=0, and at z=2H,

If ū is assumed to be a product of some function of z and t, it may be represented by the following expression: $\bar{u} = f_1(z). f_2(t)$ (49)

Where, $f_1(z) = \frac{1}{2\pi i} \oint \sum_{n=0}^{\infty} \frac{(z-z_0)^n . f(z').dz'}{(z'-z_0)^{n+1}}$ (50)

Equation of consolidation is given by:

$$\frac{\partial \bar{u}}{\partial t} = C_v \frac{\partial^2 \bar{u}}{\partial z^2} \quad (51)$$

And above equation can be written as

$$f_1(z).\frac{\partial u}{\partial t}[f_2(t)] = c_v f_2(t).\frac{\partial^2}{\partial z^2}[f_1(z)] \qquad (52)$$

or $$\frac{\frac{\partial^2}{\partial z^2}[f_1(z)]}{f_{1(z)}} = \frac{\frac{\partial}{\partial t}[f_2(t)]}{c_v f_2(t)} \qquad (53)$$

In above equation, L.H.S does not contain t and hence it is taken as constant if t is considered variable. Similarly, the R.H.S is constant when z is considered variable. Hence each term must be equal to a constant ($-A^2$) and hence represented by the following r:

$$\frac{\partial^2}{\partial z^2}[f_1(z)] = -A^2 f_1(z) \qquad (54)$$

$$\frac{\partial^2}{\partial t^2}[f_2(t)] = -A^2.c_v.f_2(t) \qquad (55)$$

Equation (17) and (18) can be satisfied respectively by the following:

$$f_1(z) = C_1 \cos Az + C_2 \sin Az \qquad (56)$$

$$f_2(t) = C_3 e^{-A^2.c_v.t} \qquad (57)$$

Where C_1 , C_2 , C_3 = arbitrary constants; e = base of hyperbolic = 2.718 $f_1(z)$ represents function of 'z' i.e. depth with respect to pore pressure ('u') lies within domain (0,2π), such that with time it varies linearly. Equation (49) is solved by expansion through 'Laurent series' & it can be split in to two components.

$$f(z) = \frac{1}{2\pi i}\oint_{C1}\frac{f(z')dz'}{z'-z} - \frac{1}{2\pi i}\oint_{C2}\frac{f(z').dz'}{z'-z} \qquad (58)$$

Where C_1 & C_2 = Contour constants are determined by applying boundary conditions, in such a way that C_2 is transverse in the positive clockwise direction. Noting that for

C_1, $|z'-z_0|\rangle|z-z_0|$ while for C_2, $|z'-z_0|\langle|z-z_0|$ .Thus,

$$f'(z) = \frac{1}{2\pi i}\sum_{n=0}^{\infty}(z-z_0)^n\oint_{C1}\frac{f(z')dz'}{(z'-z_0)^{n+1}} + \frac{1}{2\pi i}\sum_{n=1}^{\infty}(z-z_0)^{-n}\oint_{C2}(z'-z_0).f(z')dz' \qquad (59)$$

Let R_1, $R_1 = \frac{1}{2\pi i}\sum_{n=0}^{\infty}(z-z_0)^n \oint_{C1}\frac{f(z')dz'}{(z'-z_0)^{n+1}}$ (60)

And is determined by Taylor expansion and convergent for $|z-z_0|\langle|z'-z_0|=r_1$ for all 'z'interiortolargercircle&

$R_2 = \frac{1}{2\pi i}\sum_{n=1}^{\infty}(z-z_0)^{-n}\oint_{C2}(z'-z_0).f(z')dz'$ (61)

is convergent for $|z-z_0|\rangle|z'-z_0|=r_2$ exterior to smaller parabolic domain

$C_2 \rightarrow$ counter clockwise. Now replace 'n' by 'n[1]' in S_2 in Eqn. (55) & on adding Eqn. (22) & Eqn.(23). We get $u_0 = \sum_{n=0}^{\infty} c \ sin\left(\frac{n\pi z}{h}\right).e^{tc_v.\left(\frac{n\pi z}{h}\right)^2}$ (62)

or $u_0 = \sum_{n=0}^{\infty} c.sin\left(\frac{n\pi z}{h}\right)$ (63)

Sine function of fourier series can be converted easily in to Laurent series function by expressing it in the form of $u_0 = \sum_{n=-\infty}^{+\infty} c.\left(\frac{e^{iz\frac{n\pi}{h}}-e^{-iz\frac{n\pi}{h}}}{2}\right)$ (64)

or $u_0 = \frac{1}{2}\left[\sum_0^{\infty} c_1.e^{iz\frac{n\pi}{h}} - \sum_{-\infty}^{0} c_2.e^{-i\frac{n\pi}{h}.z}\right]$ (65)

Now let us assume 'z' = $e^{ik\theta}$ and replace the value of z above with the same we get,

$u_0 = c_k\left(z^{\frac{\pi}{h}}\right)^n$ (66)

Where c_k is complex consolidation constant.Now from Laurent series, At time't', when z = 0; ū = 0.Limit changes from -∞ $to - \pi$ and from $+\infty$ to

$u_0 =$ f (z) = $\sum_{n=-\pi}^{+\pi} a_n.(z-z_0)^n$ (67)

Where $a_n = \frac{1}{2\pi i}\oint_c \frac{f(z')dz'}{(z-z_0)^{n+1}}$ (68)

Therefore, ū = $\sum_{n=-\pi}^{+\pi} a_n.(z-z_0)^n.e^{-A^2.C_v.t}$ (69)

U (%) = $\sum_{-\pi}^{+\pi}(-a_n.z_0)^n.e^{-A^2.C_v.t}$ (70)

$$U\ (\%) = \frac{u_0}{2\pi i} . \oint \frac{u_0}{\left(z^{\frac{\pi}{h}}\right)^{n+1}} \quad (71)$$

$$U\ (\%) = \sum_{-\pi}^{+\pi} \left(\frac{1}{1+z}\right)^n . e^{-A^2 . C_v . t} \quad (72)$$

$$\bar{u} = n_1 . e^{-A^2 . C_v . t} - n_2 (z . e^{-A^2 . C_v . t}) + n_3 (z^2 . e^{-A^2 . C_v . t}) + \ldots + \sum_{n=0}^{\infty} \{(-z)^n . e^{-A^2 . C_v . t}\} \quad (73)$$

Where $c_v = \frac{1}{2\pi i} \oint \frac{U_0\, dz}{\left(z^{\frac{\pi}{h}}\right)^{n+1}}$ (74)

Hence, $U\ (\%) = 1 - \frac{8}{\pi^2} \sum_{n=-\infty}^{+\infty} \{\frac{1}{(2n+1)^2} \times e^{-(2n+1)^2 . \pi^2 . T_v}\}$ (75)

And $U\ (\%) = f\ (T_v)$ (76)

Figure below shows the general consolidation pattern using Laurent series. Dissipation of pore water pressure from thin clay laminae on application of loads show similar behaviour as shown following elliptical path

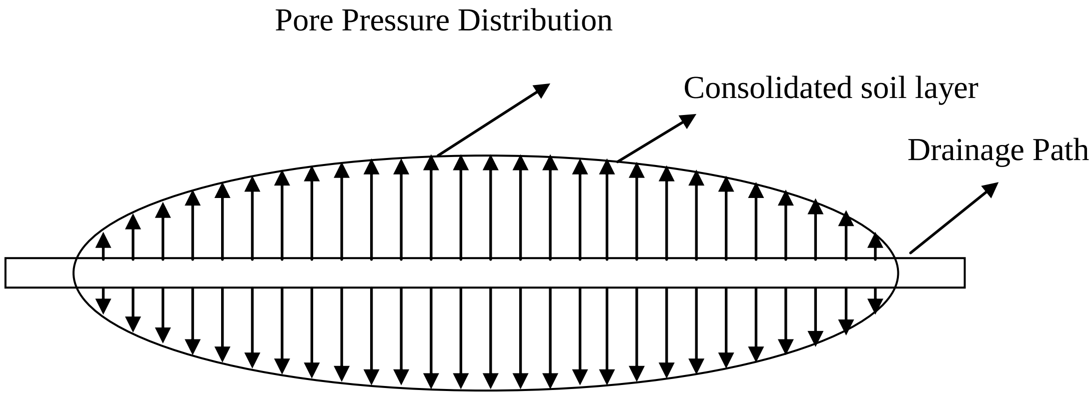

Fig 3.3: General consolidation pattern using Laurent series.

Fig 3.4 shows a thin strip of clay lamina, and progressive loads were applied as a result of which dissipation of pore water pressure started, and it took place at first from positive terrain only, as a result up to thickness 't' at the upper half (positive) domain of lamina, consolidation took place completely and soil get compacted. In the figure, it is clear that consolidation of the part of the sample that lies just beneath the load that is up to positive domain of Laurent series, is completed and nature of consolidation curve after 99% degree of consolidation converges into elliptical path from parabolic path at starting. Now in the figure shown below, Consolidation is going to be achieved completely in positive (real) and negative (imaginary) terrain / roots of Laurent series. And rate of consolidation follows same pattern in both domain that is completely elliptical and pattern of pore water dissipation in both the regions are mirror image of each other that is zero at corners and maximum at centre. Finally the pore water dissipation of thin clay lamina is completed and total consolidation can be achieved including primary and secondary consolidation from initial depth z_0 to z of thickness 't' with two circle of convergence of radius from R_1 to R_2 with two sets of contour constants C_1 and C_2.

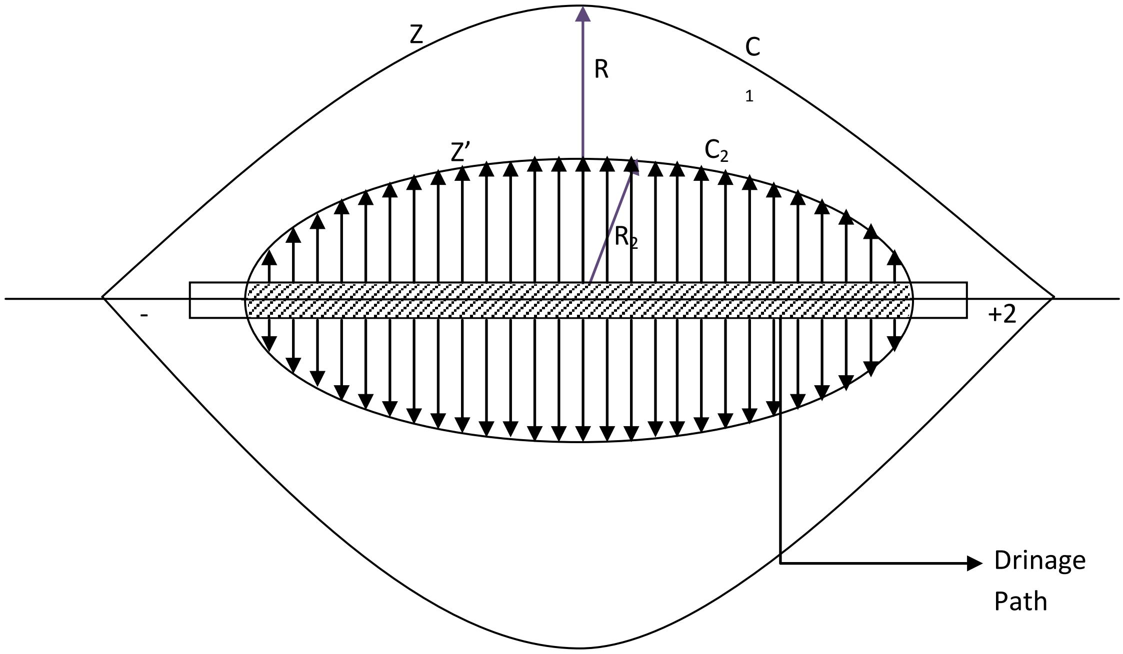

Fig 3.4: Total primary & secondary consolidation achieved

(z' = depth of first layer , z = depth of second layer)

CHAPTER – 4

EXPERIMENTAL WORKS

4.1 INTRODUCTION

The current practice in some places is to consolidate the multilayered soil found in different areas. In the present study an experimental work was conducted to evaluate the consolidation characteristics and other parameters. The index as well as engineering properties have been evaluated. Details of material used, processing test procedure adopted are described in this chapter.

The experiments which were performed for 2 different layers of soil i.e. for clay & sand.

Characterization of clay:-

1. Moisture Content determination.
2. Density bottle test to determine specific gravity.
3. Sieve analysis to find grain size distribution.
4. Liquid limit & Plastic limit determination.
5. Ash Content & loss of ignition test.
6. Swelling Index

Characterization of sand:-

1. Density bottle test to determine specific gravity.
2. Sieve analysis to find grain size distribution.
3. To determine different properties of Compaction.
4. Determination of coefficient of permeability.

4.2 EXPERIMENTAL ANALYSIS

4.2.1 For Clay Layer: -

4.2.1.1 Moisture content determination:

Sample is being taken in three different crucibles of about 10 gms. And kept in the oven for about 24 hours at 55◦C.After 24 Hours weight is again taken through weighing machine and moisture content is determined.

Table 4.1: Moisture Content Determination

SAMPLE NAME	INITIAL WEIGHT (gm)	DRIED WEIGHT (gm)	WATER CONTENT (%)
A	10	9.3	7
B	10	9.5	5
C	10	9.6	4

Average moisture content **is 5.33%**

4.2.1.2 Specific gravity by density bottle IS: 2720 (Part III)

Specific gravity is the ratio of the mass of unit volume of soil at a stated temperature to the mass of the same volume of gas-free distilled water at a stated temperature, generally taken at 4 degree centigrade.

Test procedure:

Determine and record the weight of the empty clean and dry Density bottle, **W1**. Place about 150-200 gm of a dry sample (passed through the sieve No. 10) in the Density bottle .Determine and record the weight of the Density bottle containing the dry sample, **W2**. Add distilled water to fill about half to three-fourth of the Density bottle. Soak the sample for 10 minutes. Stir the mixture rigorously with a glass rod to ensure removal of all the entrapped air. Fill the Density bottle with distilled (water to the mark),clean the exterior surface of the Density bottle with a clean dry cloth. Determine the weight of the Density bottle and contents, **W3**. Empty the Density bottle and clean it. Then fill it with distilled water only (to the mark). Clean the exterior surface of the Density bottle with a clean, dry cloth. Determine the weight of the Density bottle and distilled water, **W4**. Empty the Density bottle and clean it.

The Specific Gravity of Clay Matter was found to be **2.77**.

4.2.1.3 Particle size distribution IS: 2720 (Part IV)

There are two types of grain size analysis, first is sieve analysis and second is hydrometer analysis. The grain size analysis is widely used in classification of soils.The particle size distribution (PSD) of a powder, or granular material, or particles dispersed in fluid, is a list of values or a mathematical function that defines the relative amounts of particles present, sorted according to size. PSD is also known as grain size distribution. **Particle Size**: A better indication of the fineness is to determine the particle size distribution. For example, one can determine the mass percentage below 10 um or determine the mean particle diameter. The particle size of organic matter varies from below 75 um to 300 um or more. Thus a clay content might have the following distribution (on a mass basis): 0.7-0.8 % below 75 um, 0.5 % finer than 150 um, 35-40 % above 300 um and 60-65 % above 600 um.

- The percentage of sample retained on each sieve shall be calculated on the basis of total weight of sample retained in sieve.
- Cumulative percentage of soil retained on successive sieve is found.

4.2.1.4 Liquid limit and plastic limit determination

4.2.1.4 (a) Liquid limit determination

Theoretical background:

Liquid limit is the minimum water content at which the soil is still in liquid state but has a small shearing strength against flowing. In other words it is the water content at which soil suspension gains an infinitesimal strength from zero strength. In the standard liquid limit apparatus from practical purposes, it is the minimum water content at which part of soil cut by groove of standard dimensions, will flow together for as a distance of 12 mm (1/2 inch) under an impact of 12 blows.

Calculations:

Plot the flow curve on a semi log graph water content as the ordinate and no. of blows as abscissa. The water content corresponding to 25 blows is taken as the liquid limit of the soil.

Results:

Liquid limit ^{W}L (from graph) $= \frac{W_1 - W_2}{\log_{10}\frac{n_1}{n_2}}$

Flow index or slope of the curve, I_f (from graph)

W_1 = water content corresponding to blow n_1

W_2 = water content corresponding to blows n_2

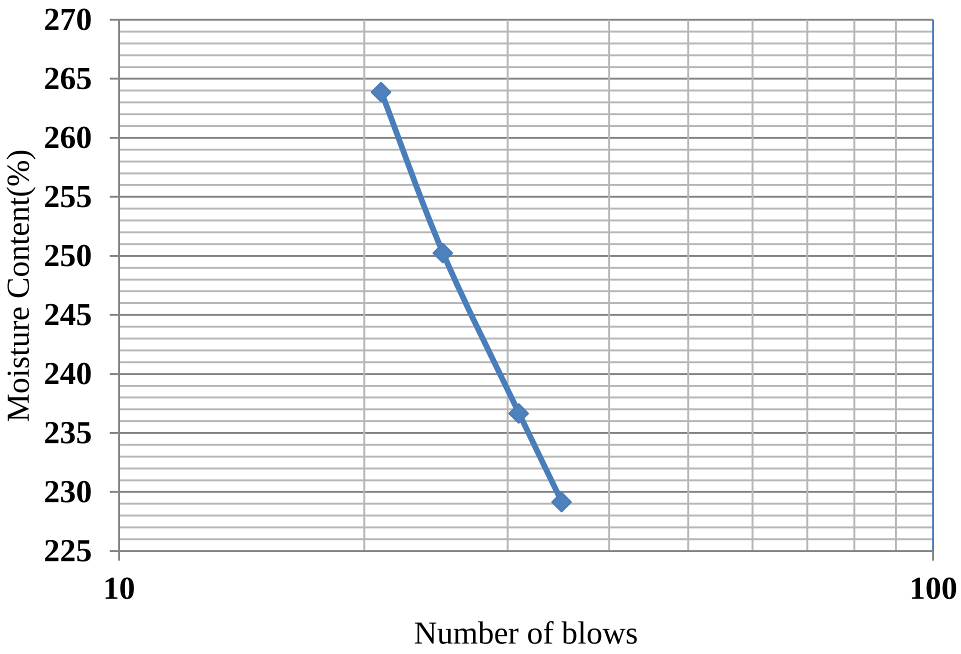

Fig 4.1.: Liquid limit determination

Liquid Limit of clay layer = **247.36%**.

4.2.1.4 (b) Plastic limit determination

Plastic limit is the minimum water content at which a soil just deigns to crumble when rolled into a thread of 3 mm in diameter. This water content in is between the plastic and semi-soil states of soil.

Thus Plastic Limit of given Clay layer is **78.6%**

4.2.1.5 Swelling index of samples

IS: 2720 {Part XL (40)} 1977.

Free Swell Index is the increase in volume of a granular material, without any external constraints, on submergence in water.

Calculations

Free Swell Index, (%) = $(V_d - V_k)/V_k \times 100\%$

V_d = Volume of the specimen read from the graduated cylinder containing Distilled water.

V_k = Volume of the specimen read from the graduated cylinder containing Kerosene

Free Swell Index is calculated as **67.4%.**

4.2.2 Characterization of Sand Layer:

4.2.2.1 Specific gravity by density bottle IS: 2720 (Part III)

Specific gravity is the ratio of the mass of unit volume of soil at a stated temperature to the mass of the same volume of gas-free distilled water at a stated temperature, generally taken at 4 degree centigrade.

.And the specific gravity of used sand sample is **2.65**.

4.2.2.2 Sieve Analysis to find grain size distribution

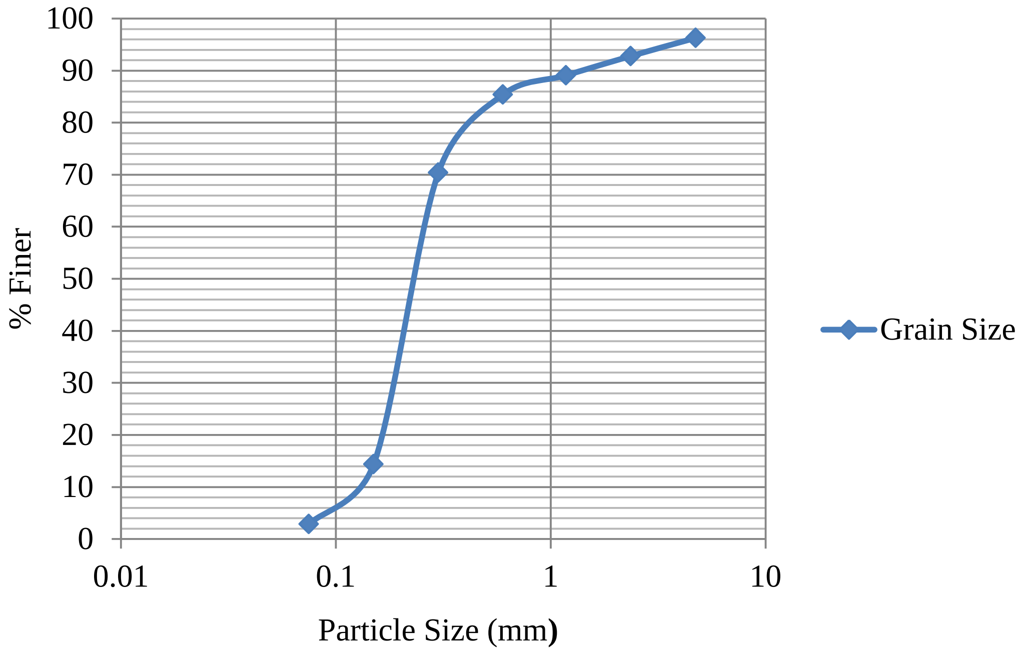

Fig 4.2: Grain size analysis of sand layer

Following terms were determined with the help of the curve in Fig 4.3 of sieving like

(a) . Effective Diameter or Effective Size $\mathbf{D_{10} = 0.14}$

(b) . D_{30} & D_{60} , $\mathbf{D_{30} = 0.19}$, $\mathbf{D_{60} = 0.26}$

(c) . C_u & C_c , $\mathbf{C_u}$ = (D_{60}/D_{10}) = **1.857** , $\mathbf{C_c}$ = $[(D_{30})^2/(D_{60}) \text{ X } (D_{10})]$ = **0.992**

(d) . Nomenclature of Sand sample is silty sand.

4.2.2.3 Determination of compaction properties of sand layer by Standard proctor test IS 2720(VII):1980

The standard proctor test was invented by R.R.Proctor (1933) for the construction of earth fill dams in the state of California.The bulk density and the corresponding dry density for the compacted soil are calculated from this.The test is repeated with increasing water contents, and the corresponding dry density obtained is therefore determined. A compaction curve is plotted between the water content as abscissa and the corresponding dry densities as ordinates. The dry density goes on increasing till the maximum density is reached. This density is called maximum dry density (MDD) and the corresponding moisture content is called optimum moisture content (OMC).

Following terms were determined with the help of the curve in Fig 4.4 of compaction

(a) .Maximum Dry Density of given sand layer is **1.89 g/cc.**

(b) .Optimum Moisture Content (OMC) of sand layer is **13.5%**

(c) .Void Ratio of Compacted sand layer = e = $(G.\gamma_w/\gamma_d)$ -1 =0.508 = **51%**

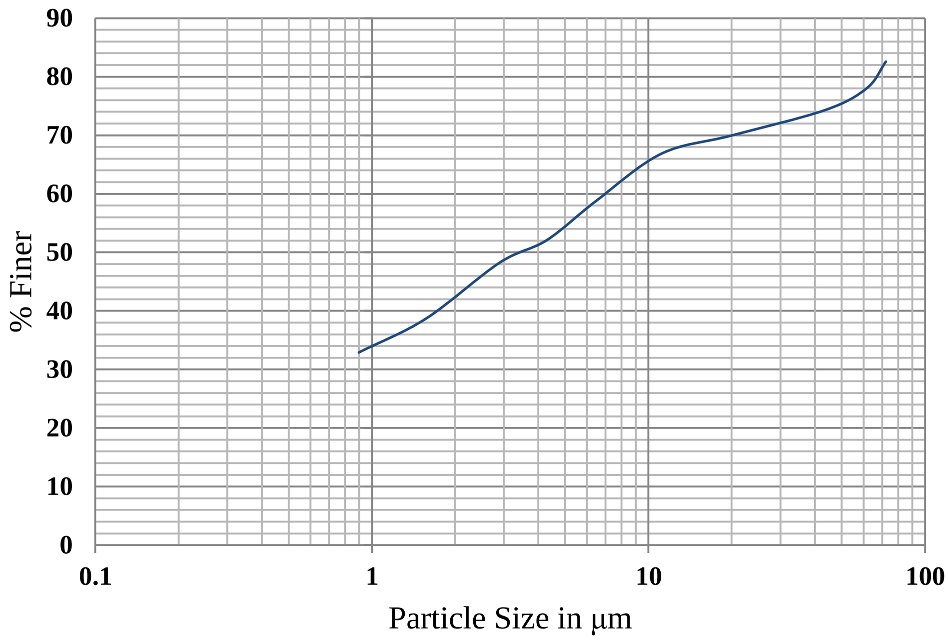

Fig 4.3 Idealised Grain distribution of clay.

4.2.2.4 Determination of coefficient of permeability

Theoretical Background:

The property of material which permits fluids to percolate through its voids is called permeability. According to Darcy's law in the laminar range the velocity of percolation is proportional to the hydraulic gradient. $V \propto i$

V= Ki

AV =Kai

AV= flow rate = Q= Kai

$$K = \frac{QL}{hA}$$

By Constant Head permeability method, Coefficient of permeability 'k' for this silty sand is determined and is about **4.26X10^{-5}** cm/sec.

4.3 CONSOLIDATION BY ODEOMETER

Consolidation is determined by using Odeometer after preparing the sample. Specification of apparatus:

i. Consolidometer consisting essentially;
   a. A ring of diameter = 60mm and height = 20mm
   b. Two porous plates or stones of silicon carbide, aluminium oxide or porous metal.
   c. Guide ring.
   d. Outer ring.
   e. Water jacket with base.
   f. Pressure pad.
   g. Rubber basket.

i. Loading device consisting of frame, lever system, loading yoke dial gauge fixing device and weights.

ii. Dial gauge to read to an accuracy of 0.002mm. .

iii. Stopwatch to read seconds.

iv. Sample extractor.

v. Miscellaneous items like balance, soil trimming tools, spatula, filter papers, sample containers.

Procedure

- Sample is prepared by using thin clay laminae in mould and then it is used in consolidation.
- Saturate two porous stones either by boiling in distilled water about 15 minute or by keeping them submerged in the distilled water for 4 to 8 hrs. Wipe away excess water. Fittings of the consolidometer which is to be enclosed shall be moistened.
- Assemble the consolidometer, with the soil specimen and porous stones at top and bottom of specimen, providing a filter paper between the soil specimen and porous stone. Position the pressure pad centrally on the top porous stone.
- Mount the mould assembly on the loading frame, and center it such that the load applied is axial.
- Position the dial gauge to measure the vertical compression of the specimen. The dial gauge holder should be set so that the dial gauge is in the begging of its releases run, allowing sufficient margin for the swelling of the soil, if any.
- Connect the mould assembly to the water reservoir and the sample is allowed to saturate. The level of the water in the reservoir should be at about the same level as the soil specimen.
- Apply an initial load to the assembly. The magnitude of this load should be chosen by trial, such that there is no swelling. It should be not less than 50 g/cm2 (5 kN/m2) for ordinary soils & 25 g/cm2 (2.5 kN/m2) for very soft soils. The load should be allowed to stand until there is no change in dial gauge readings for two consecutive hours or for a maximum of 24 hours.
- Note the final dial reading under the initial load. Apply first load of intensity 0.1 kg/cm2 (10kN/m2) start the stop watch simultaneously. Record the dial gauge readings at various time intervals (and fill in the

table). The dial gauge readings are taken until 90% consolidation is reached. Primary consolidation is gradually reached within 24 hrs.

- At the end of the period, specified above take the dial reading and time reading. Double the load intensity and take the dial readings at various time intervals. Repeat this procedure fir successive load increments.
- The usual loading intensity are as follows: 0.1, 0.2, 0.5, 1, 2, 4 and 8 kg/cm2.
- After the last loading is completed, reduce the load to half (1/2) of the value of the last load and allow it to stand for 24 hrs. Reduce the load further in steps of 1/4th the previous intensity till an intensity of 0.1 kg/cm2 is reached. Take the final reading of the dial gauge.
- Quickly dismantle the specimen assembly and remove the excess water on the soil specimen in oven, note the dry weight of it.

After getting the required data e-log ρ curve for each set of three samples is plotted as follows:

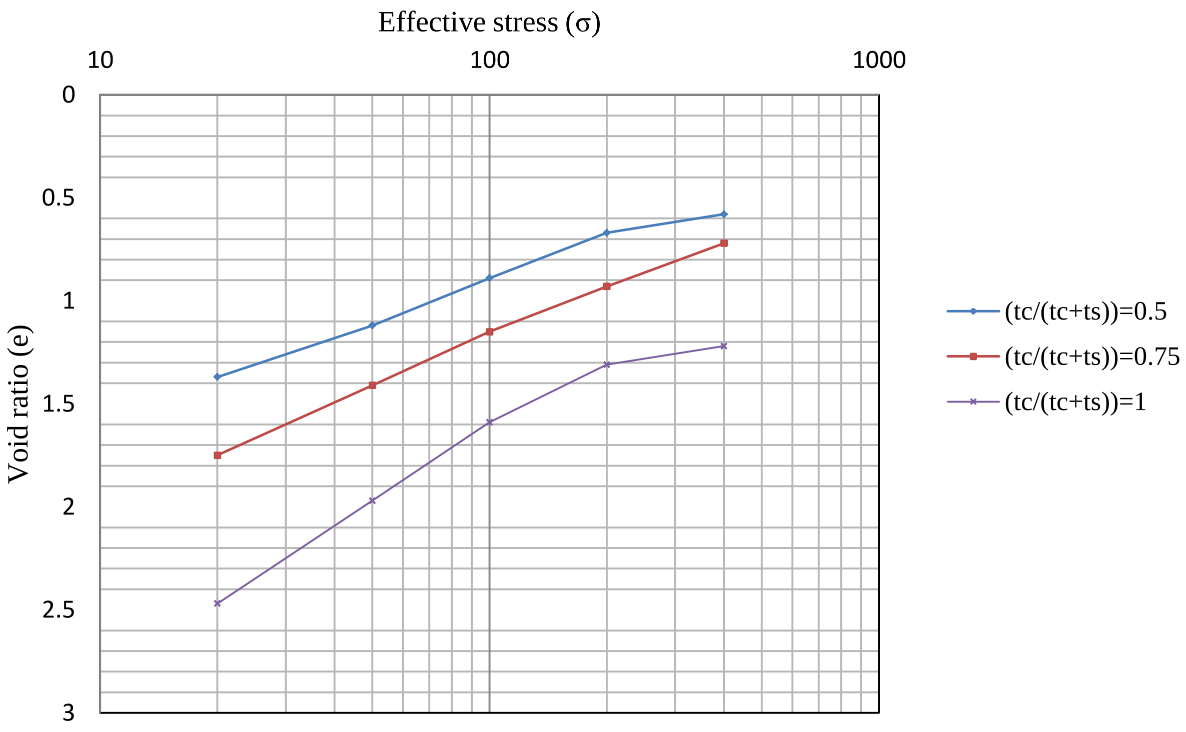

Fig 4.4: e-log ρ curve

Compression index C_c can now be calculated from the above graph

for all the three sets of samples. Thus, C_c

(a). For pure clay of thickness 2cm, $\frac{tc}{(tc+ts)}$ = 1.0 is 0.448.

(b). For 1.5 cm clay & 0.5 cm sand layer , $(\frac{tc}{(tc+ts)})$ = 0.75 is 0.365.

(c). 1cm thick clay layer & 1 cm thick sand layer. $(\frac{tc}{tc+ts})$ = 0.5 is 0.205.

Result of Consolidation Test:

(a) . For pure clay of thickness 2cm, $\frac{tc}{(tc+ts)}$ = 1.0

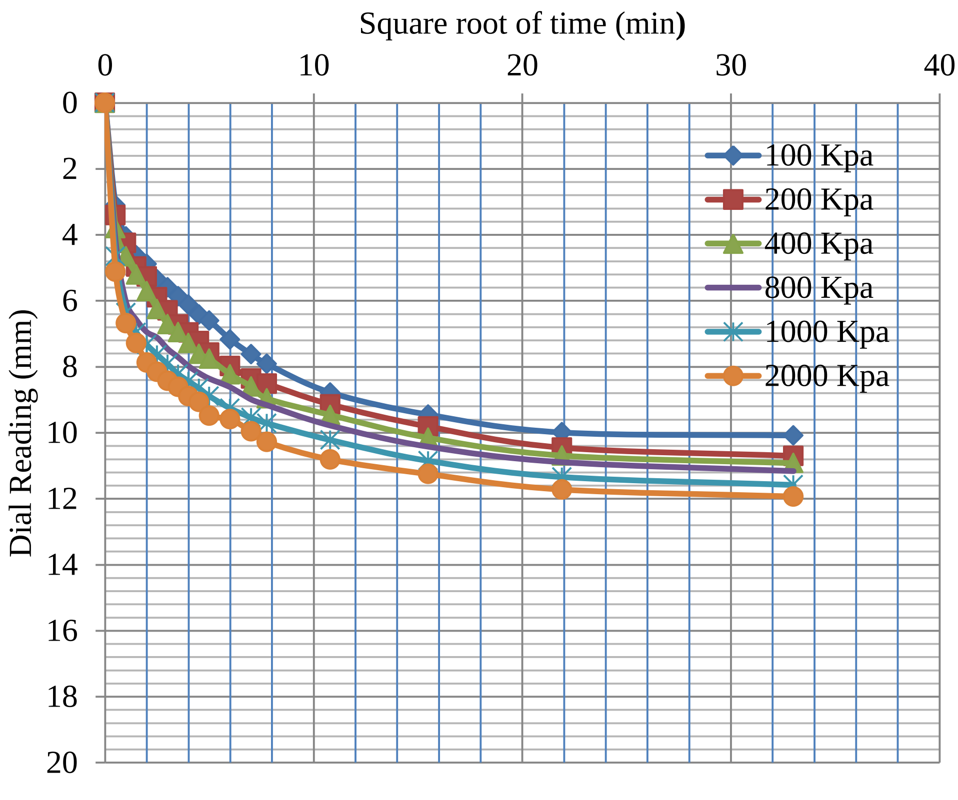

Fig 4.5: Rate of consolidation for $S_{1.00}$

Squareroot of time fitting method ($\sqrt{T_v}$ & U) is calculated from the equation.

- Coefficient of Consolidation C_v =2.261 X 10^{-5} cm^2/sec.
- Compression Index: Cc =0.448.

- Coefficient of compressibility: a_v=5.97 x $10^{-4}m^2$/KN
- Coefficient of Volume Compressibility = 4.1209x$10^{-4}m^2$/KN

After taking the layer of sand and clay in consecutive layers of varrying thickness such that thickness of clay layer decreases & we study the behaviour of degree of consoliation and speed of consolidation of the given soil strata.

(b) - For 1.5 cm clay & 0.5 cm sand layer , $\left(\frac{tc}{(tc+ts)}\right) = 0.75$

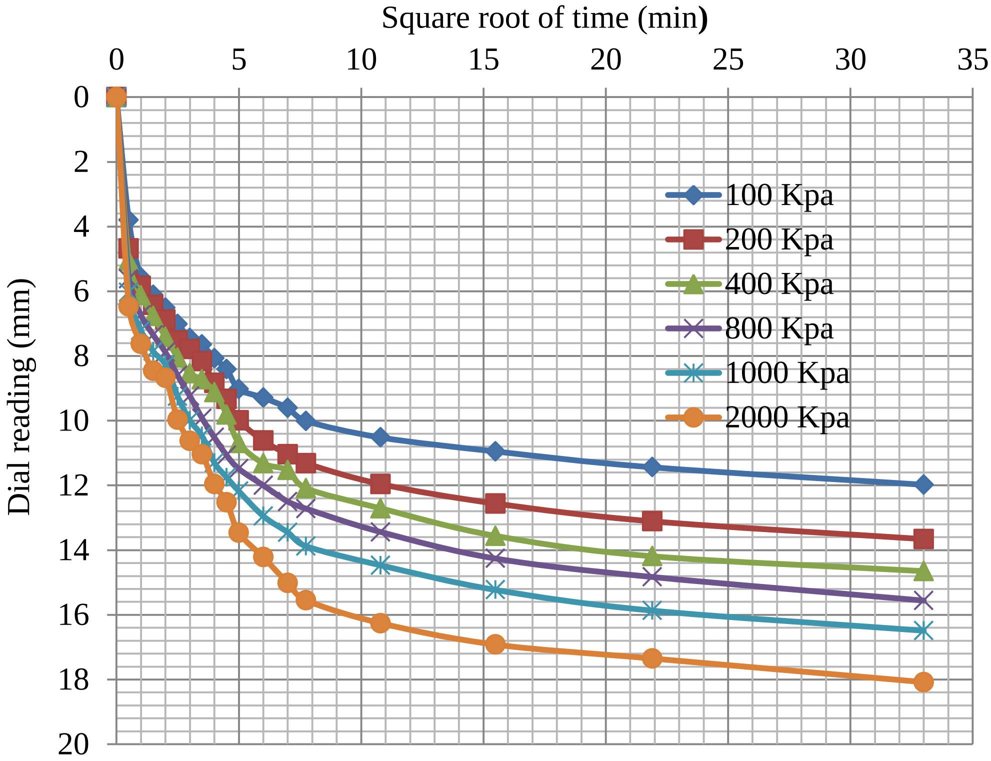

Fig 4.6: Rate of consolidation for $S_{0.75}$

Squareroot of time fitting method ($\sqrt{T_v}$ & U) is calculated from the equation.

- Coefficient of Consolidation C_v =2.61 X 10^{-5} cm^2/sec.
- Compression Index: Cc =0.365.
- Coefficient of compressibility: a_v=4.677 x $10^{-4}m^2$/KN
- Coefficient of Volume Compressibility = 3.1204x$10^{-4}m^2$/KN

After taking the layer of sand and clay in consecutive layers of varrying thickness such that thickness of clay layer decreases & in order to study the behaviour of degree of consoliation and speed of consolidation of the given soil strata. Now increasing the thickness of clay layer i.e

(b) **. 1cm thick clay layer & 1 cm thick sand layer.** $(\frac{tc}{tc+ts})$ = 0.5

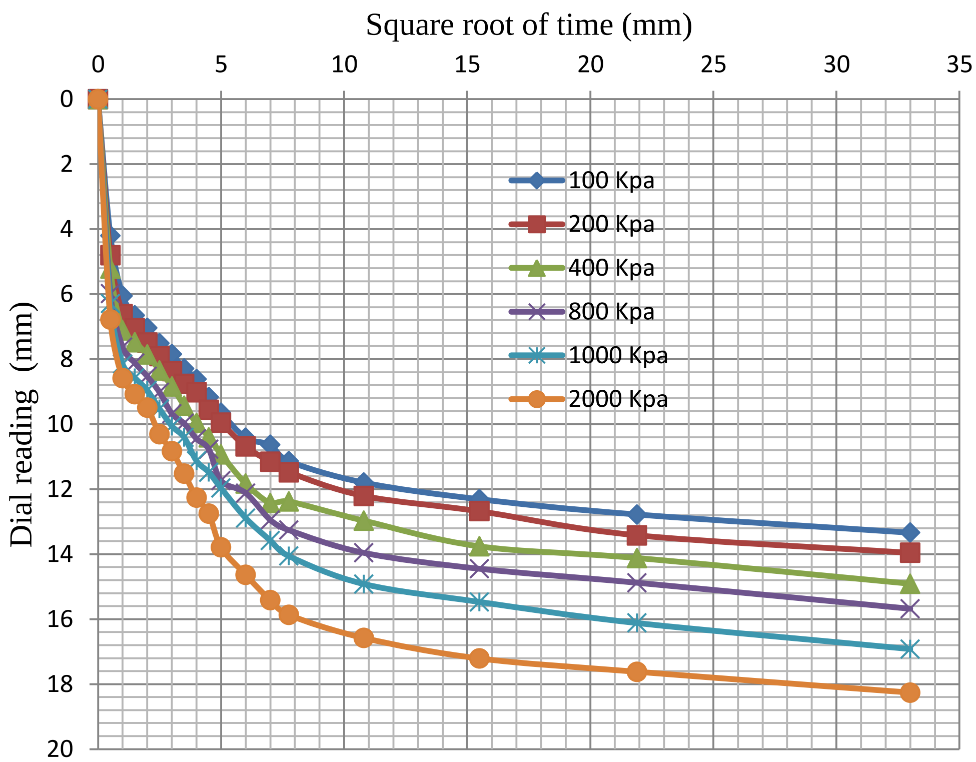

Fig 4.7: Rate of consolidation for $S_{0.5}$

Squareroot of time fitting method ($\sqrt{T_v}$ & U) is calculated from the equation.

- Coefficient of Consolidation C_v =4.205 X 10^{-5} cm^2/sec.
- Compression Index: Cc =0.205.
- Coefficient of compressibility: a_v=3.329 x $10^{-4}m^2$/KN
- Coefficient of Volume Compressibility = 2.0419x$10^{-4}m^2$/KN

Table 4.2: Test programme and composition of clay-sand layers used in oedometer test

Nomenclature	No. of samples	Thickness of clay layer, tc (mm)	Thickness of sand layer, ts (mm)	$tr = \frac{tc}{(tc+ts)}$
$S_{1.00}$	3	20	0	1
$S_{0.75}$	3	15	5	0.75
$S_{0.50}$	3	10	10	0.5

Table 4.3: Results of consolidation test on varied composition

Nomenclature	tr	Cv (mm^2s^{-1})	Cc	a_v (m^2kN^{-1})	mv (m^2kN^{-1})
$S_{1.00}$	1	$2.261 \text{ X } 10^{-3}$	0.448	$5.97 \text{ x } 10^{-4}$	$4.1209\text{x}10^{-4}$
$S_{0.75}$	0.75	$3.305\text{X}10^{-3}$	0.365	$4.67\text{x}10^{-4}$	$3.1204\text{x}10^{-4}$
$S_{0.50}$	0.5	$4.205 \text{ X } 10^{-3}$	0.205	$3.329 \text{ x } 10^{-4}$	$2.0419\text{x}10^{-4}$

Fig 4.8: Parts of Consolidometer, loose

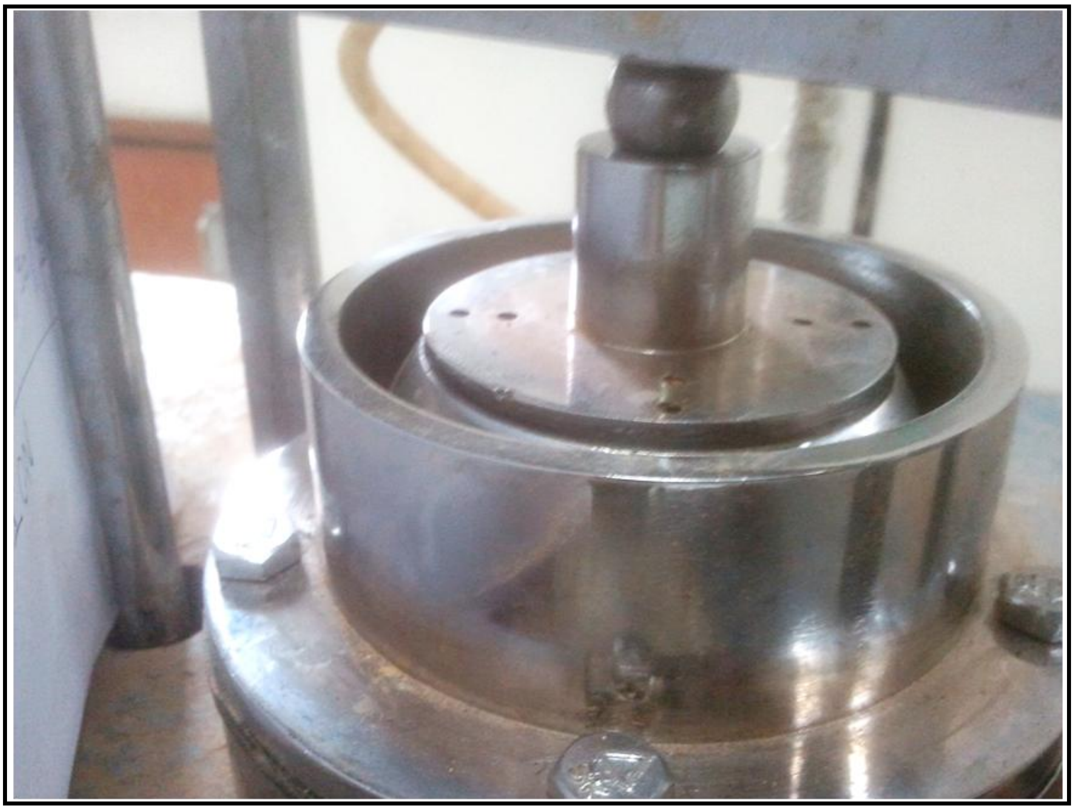

Fig 4.9: Parts of Consolidometer assembled

Fig 4.10: Consolidation test apparatus.

CHAPTER-5

RESULTS AND VALIDATION

5.1 RESULTS OF NUMERICAL TECHIQUE

With the help of Laurent series one can get faster rate of dissipation of pore water pressure from thin clay laminae,and about 99% of total consolidation is achieved unlike to 90% of consolidation from terzaghi analysis using fourier series.Different sets of formulation were obtained with constant boundary conditions as taken by Tezaghi using Fourier series,but in results of both a clear variation is visible, and it is clear that rate of dissipation of pore pressure in thin clay laminae achieved by laurent series occurs in faster rate as compared to fourier series.Nature of curve obtained from Fourier series is logarithmic in nature while that obtained from Laurant series is exponential series ofhigher order.

5.1.1 Theoritical Comparison of 2 Series

Table 5.1: Comparisons of two models (Theoretically)

S.No	Fourier Series	Laurent Series
(a)	Series not approaches to completeness.	Series approaches to completeness.
(b)	Series varies with in the domain (0,2π).	Series varies with in the domain (-∞,+∞).
(c)	Basic equation of Fourier series f(z) $= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n.\cos nz + \sum_{n=1}^{\infty} b_n.\sin nz$ Where $a_n = \frac{1}{\pi}\int_0^{2\pi} f(z).\cos nz\,.dz$ $b_n = \frac{1}{\pi}\int_0^{2\pi} f(z).\sin nz\,.dz$	Basic equation of Laurent series were in the form of f(z) = f(z) $=\sum_{-\infty}^{\infty} a_n.(z-z_0)^n$

S.No	Fourier Series	Laurent Series
(d)	Fourier Series can handle discontinuities between the limits of the function.	Laurent Series cannot handle discontinuities between the limits of the function
(e)	• Application Of Series : • Representation of High frequency square & sawtooth wave. • Full – wave rectifier • Degree of consolidation • Thermo elastic problems	Application Of Series : • Representation of hydel waves with Bernaulli equation • Representation of Infinite series • Degree of consolidation

5.1.2 Computation of Degree of Consolidation Using Laurent Series

Degree of consolidation is determined with the help of both fourier and laurent series,as from different theories it is clear that Terzaghi had used only real roots that is positive roots of the initial pore water pressure and ignore all the negative roots as a result, results of the analysis is not perfectly true.While Laurent series incorporates both positive as well as negative root resembling contractive and dilative behaviour of thin clay laminae under application of loads.

Different mathematical relations so obtained after determining rate of consolidation using laurent series were: as degree of consolidation obtained using Eqn. (75).

$$U (\%) = 1- \frac{8}{\pi^2} \sum_{n=-\infty}^{+\infty} \{\frac{1}{(2n+1)^2} \times e^{-(2n+1)^2 . \pi^2 . T_v}\}$$

From the above formula, rate of consolidation is computed at different degrees using curve fitting method and following results were tabulated.

Table 5.2: Relationship for time factor using real roots

U	Fourier transform	Terzaghi analysis
~ 0.53	$\frac{\pi}{4}\ U^2$	$\frac{\pi}{4}\ U^2$
>0.53	$1.781 - 0.933\{\log(1-U)+2\}$	$1.781 - 0.933\{\log(1-U)+2\}$
Terzaghi had used Fourier series with real roots and hence both of them are same.		

Table 5.3: Relationship for time factor using real and imaginary roots

U	Fourier transform	Laurent Transform
~ 0.10	$\frac{\pi}{4}\ U^2$	$\frac{\pi}{14}U^5$
0.1 to 0.3	$\frac{\pi}{8}U^5$	$\frac{\pi}{8}U^5$
0.3 to 0.9	$\frac{\pi}{8}U^5$	Stirling's Formula*

*$T_v = \frac{1}{(U+1)!} \times \sqrt{2\pi}U^{U+\frac{1}{2}} \times e^{-U}(1+\frac{1}{12}U)$ [Afrken & Weber (1970)]

Stirling's formula Stirling's formula was at first invented by Abraham D. E. Moivre and printed in "Miscellanea Analytica" in 1730. It had been later refined, however within the same year by James Stirling in "Method us Differentials" beside alternative properties, it gives fabulous results. For example, stirling computes the area under the bell shaped curve. For computation of ln (z) for very large z (statistical mechanics) and for numerical computations at non integral values of z, a series enlargement of ln(z) in negative powers of z develops increased interest amongst researchers. Perhaps the foremost elegant manner of explanation such associate in nursing enlargement is by technique of steepest descents. The top of formula has a beginning with a numerical integration formula, and doesn't need information of contour integration and is especially direct. And thus once application of Laurent transform, so as to work out degree of consolidation of extremely merging series Stirling formula owe to be most fitted computation for curve fitting and thus we tend to used it.

As in present context, after the application of Laurent series for determining degree and rate of consolidation curve fitting is utmost important in order to determine different ranges and domains at which particular degree of consolidation variates with space and time. Stirling formula is the most significant and nearest similar curve fitting computation that can be used here.The basic formula of Stirling formula. If ‘S’ varies from 1 to 10, then t = value of convergence.

Thus t = $\frac{1}{(S+1)!} \times \sqrt{2\pi}S^{S+\frac{1}{2}} \times e^{-S}(1+\frac{1}{12}S)$ is computed.

5.1.3 Graphical Comparison of rate of consolidation using different series

In terms of rate of consolidation, comparison is made between two series explaining variation of ‘degree of consolidation’ with ‘time factor of given soil strata. Variation thus shows that:

Table 5.4: Variation of Degree of Consolidation (U) with Time Factor (T_v) using different transforms.

Tv	U by Laurent series	U by Fourier series with real roots	U by Fourier series with both roots
0.003	0.09923	0.0992370000	0.099235
0.004	0.099254	0.0992324500	0.099342
0.005	0.099452	0.0994410000	0.099448
0.006	0.101851	0.1016995000	0.1018
0.007	0.13289	0.1054200000	0.10625
0.008	0.157223	0.1068900000	0.11079
0.009	0.180778	0.1099675000	0.12078
0.01	0.204776	0.1118800000	0.13132
0.02	0.318301	0.1586430000	0.19732
0.03	0.390056	0.1945254490	0.25264
0.04	0.450452	0.2247700000	0.2928
0.05	0.503332	0.2514265430	0.33328
0.06	0.550492	0.2755208240	0.371968
0.07	0.592914	0.2976777920	0.416574
0.08	0.631226	0.3183009330	0.44976
0.09	0.665887	0.3376703630	0.476352
0.1	0.697273	0.3559900000	0.509604
0.2	0.887063	0.5033320000	0.714824

Tv	U by Laurent series	U by Fourier series with real roots	U by Fourier series with both roots
0.3	0.957865	0.6125520000	0.798567
0.4	0.98428	0.6972720000	0.857112
0.5	0.994135	0.7634150000	0.899612
0.6	0.997812	0.8151010000	0.935986
0.7	0.999184	0.8554940000	0.962278
0.8	0.999695	0.8870630000	0.981345
0.9	0.999886	0.9117350000	0.98932
1	0.999958	0.9310180000	0.995346
2	0.999999	0.9941350000	0.999659
5	1	0.9999963960	0.999999
10	1	1.0000000000	1

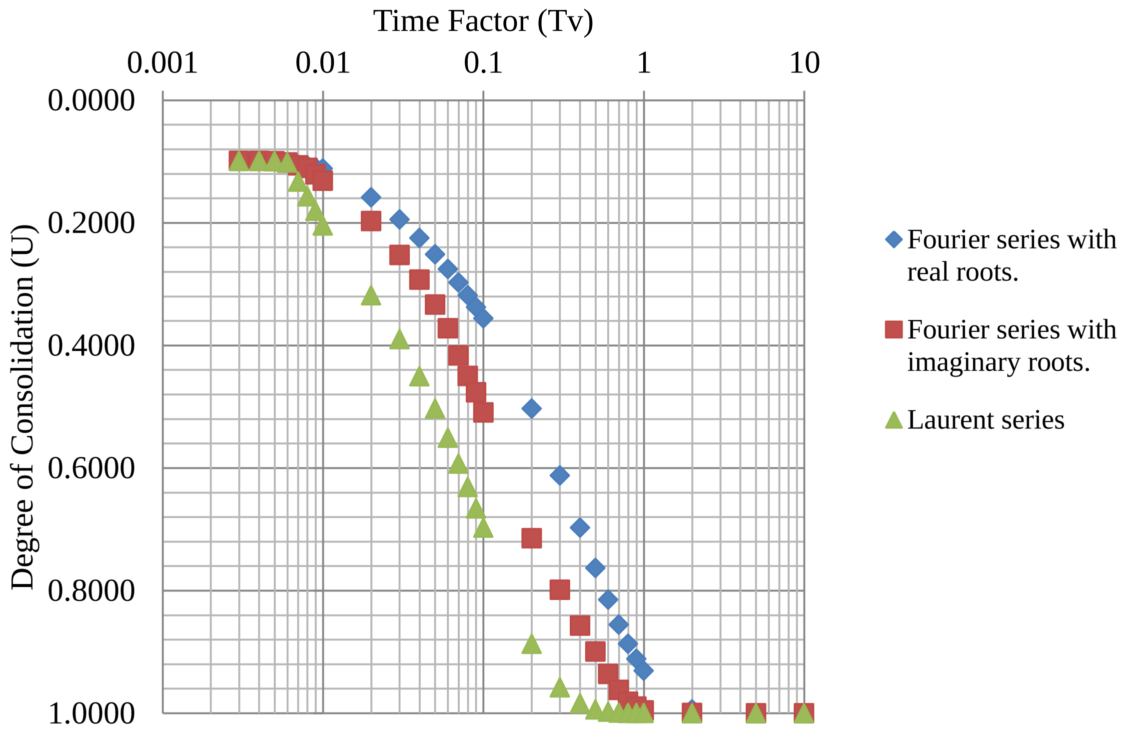

Fig 5.1: Theoretical rate of consolidation

5.2 Experimental results

Consider consolidation test using 60 mm diameter and 20mm thick sample. We used three sets of arrangements of internally varying thickness of a clay and sand in odeometer test. The clay and sand are tested for determination of their index properties. The rate of consolidation is determined with the root of time fitting method.

The clay sample is prepared in to a cylindrical saturation bracket of a diameter of 200mm and thickness of 100 mm in a water bath. A clay lamina is cut to the size of by pressing the odeometer ring in to it up a desired thickness consolidation. The saturated porous stones are processed by boiling in distilled water about 15 minute and by keeping them submerged in the distilled water for 4 to 8 hours. The fittings to be enclosed are moistened. The soil specimen in the ring is placed with a filter paper separating soil specimen and porous stone and porous stones at top and bottom of specimen.

The position of the pressure pad and the loading frame is kept central to the assembly so that the load applied is axial. After the last load increment is completed it is reduced to the half of the value and allowed to stand for 24 hours. Reduce the load further in steps of the previous intensity till an intensity of 10 kPa is reached. After taking the final reading of the dial gauge, reduce the load to the initial value keeping it for 24 hours and noted the final readings of the dial gauge. Quickly dismantle the specimen assembly and remove the excess water on the soil specimen in oven, note the dry weight of it.

Table 5.5: Test programme and composition of clay-sand layers used in oedometer test

Nomenclature	No. of samples	Thickness of clay layer, tc (mm)	Thickness of sand layer, ts (mm)	$tr = \frac{tc}{(tc+ts)}$
$S_{1.00}$	3	20	0	1
$S_{0.75}$	3	15	5	0.75
$S_{0.50}$	3	10	10	0.5

Results of test performed for classification of clayey soil indicate that it has specific gravity 2.77, moisture content 5.5%, liquid limit 247.36%, plastic limit 78.6% and swelling index 67.4%. The sand has specific gravity of 2.65, grain size distribution (D_{10} = 0.14, D_{30} = 0.19, D_{60}= 0.26, C_u = 1.857, C_c = 0.992) and void ratio of 0.51. The coefficient of permeability of sand by falling head method is found to be 4.26×10^{-2} mms^{-1}.

Table 5.6: Results of consolidation test on varied composition

Nomenclature	*tr*	Cv (mm^{2}s^{-1})	Cc	a_v (m^{2}kN^{-1})	mv (m^{2}kN^{-1})
$S_{1.00}$	1	2.261 X 10^{-3}	0.448	5.97 x 10^{-4}	4.1209x10^{-4}
$S_{0.75}$	0.75	3.305X10^{-3}	0.365	4.67x10^{-4}	3.1204x10^{-4}
$S_{0.50}$	0.5	4.205 X 10^{-3}	0.205	3.329 x 10^{-4}	2.0419x10^{-4}

Also from the test performed in fixed type odeometre under numerous loads for determining the rate of consolidation with different layers of varrying thickness of clay & sand strata , it is clear that 'rate of consolidation' $C_{v\,,}$ increases with the increasing thickness of sand layer which is to be sandwiched beneath clay layer and a clear comparison is made between all the three sets of sample for say 2 loads.It can be clearly seen from graph.

1. For σ = 100 Kpa,

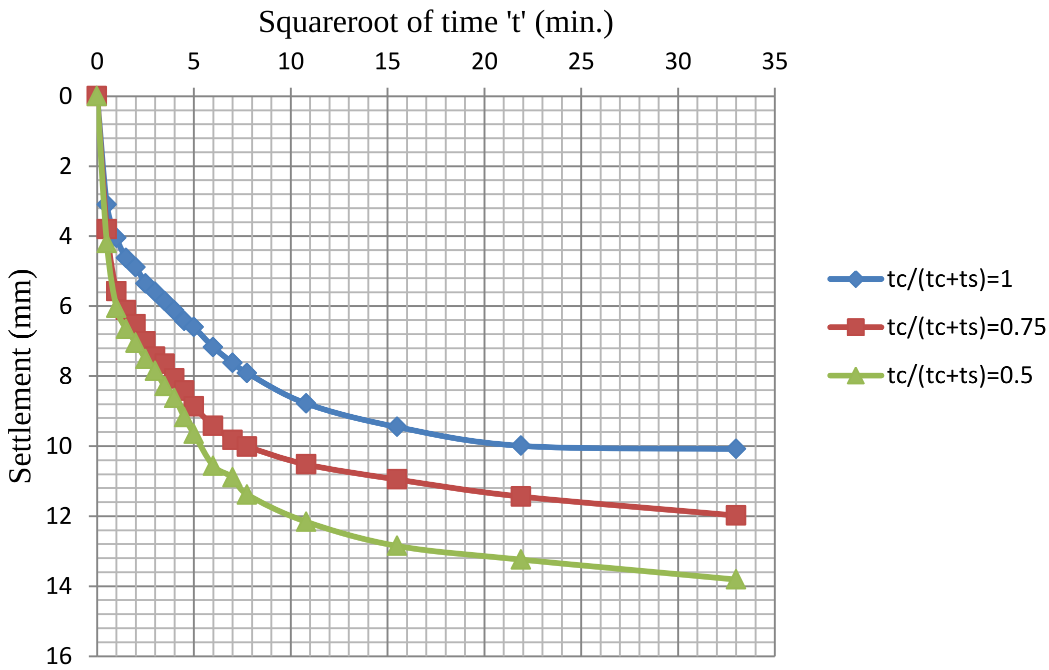

Fig 5.2: Comparison of results

2. For σ = 2000 Kpa

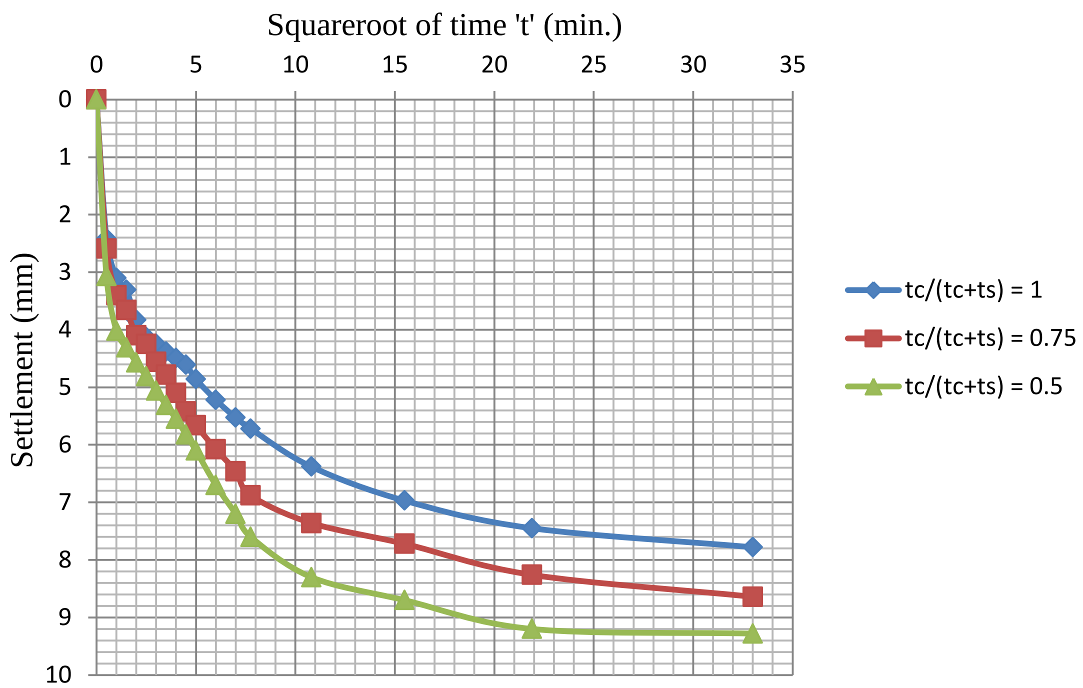

Fig 5.3: Comparison of results

Thus from above curve it is clear that as the thickness of clay layer is reduced by increasing the thickness of layer of sand rate of consolidation increases sharply.

1. Rate of Consolidation U_{90}(%) is going to be increased.
2. Coefficient of cosolidation Cv obtained increases with increase in the applied pressure.
3. t_{90} i.e. time required to achieve 90% consolidation we get

1. For σ = 10Kpa

Table 5.7: Determination of Consolidation Properties

$t_c/(t_c+t_s)$	√t 90 (min.)	t 90(Hrs.)	C_v (cm²/sec)
1	22.5	8hrs 27minutes	2.261 X 10^{-5}

$t_c/(t_c+t_s)$	**√t 90 (min.)**	**t 90(Hrs.)**	**C_v (cm^2/sec)**
0.75	21	7hrs 21minutes	2.6 X 10^{-5}
0.5	16.5	4hrs 33minutes	4.205 X 10^{-5}

2.For σ = 200Kpa

Table 5.8: Determination of Consolidation Properties

$t_c/(t_c+t_s)$	**√t 90 (min.)**	**t 90(Hrs.)**	**C_v (cm^2/sec)**
1	20.8	7hrs 13minutes	2.65 X 10^{-5}
0.75	19.1	6hrs 5minutes	3.14 X 10^{-5}
0.5	14.5	3hrs 31minutes	5.445 X 10^{-5}

5.3 Validation of numerical technique with expermental analysis

The dissipation of the pore water pressure of a thin clay lamina with time and space can be analysed using Fourier and Laurent transform. This study shows that the results of Fourier and Laurent transform converges sharply ignoring the imaginary roots of the general solution. Terzaghi utilized the results of the positive roots of the Fourier transform.Results were analysed with both real and imaginary roots using

Fourier and Laurent transform. The real and the imaginary roots of the Laurent transform together satisfy the dissipation of pore water in contractile and dilatant regime. While the results of real roots present only the solution to the dissipation of pore water pressure in elastic regime. The experimental observations of the dissipation of pore water pressure of silty sand converge with the results of the solution obtained from the solutions of the Laurent transform and hence validate this proposal. Following graphs provide better visualization for above said proposal:

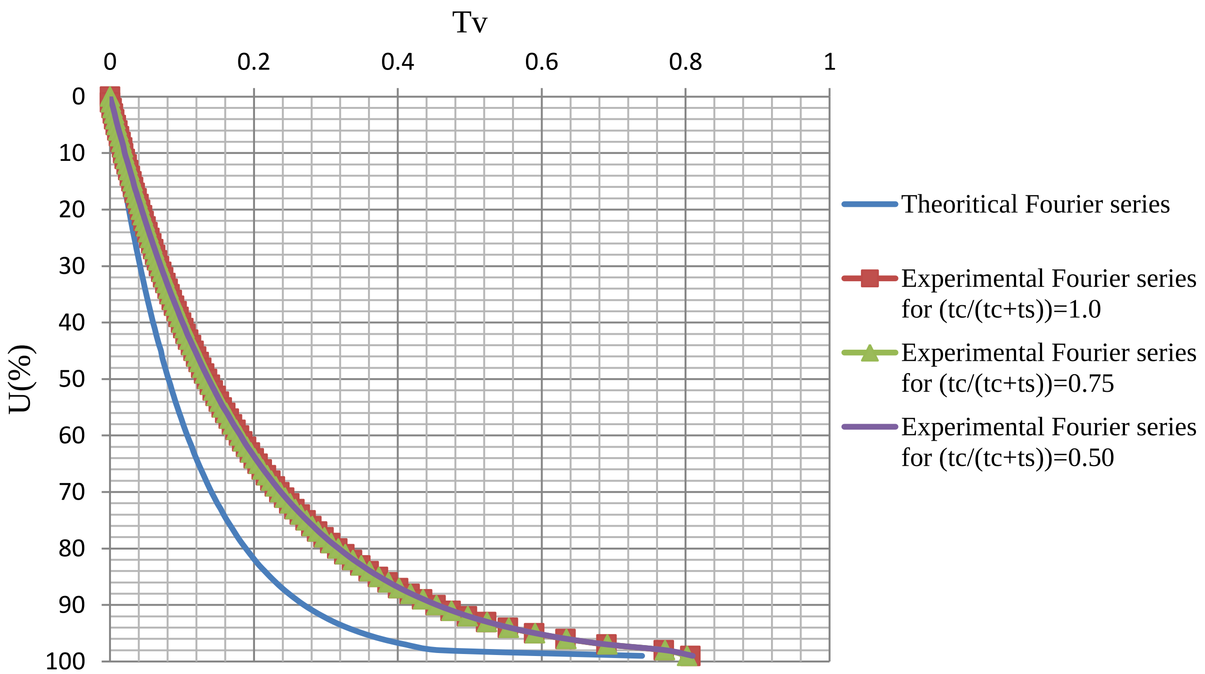

Fig 5.4: Variation of degree of consolidation with Fourier series.

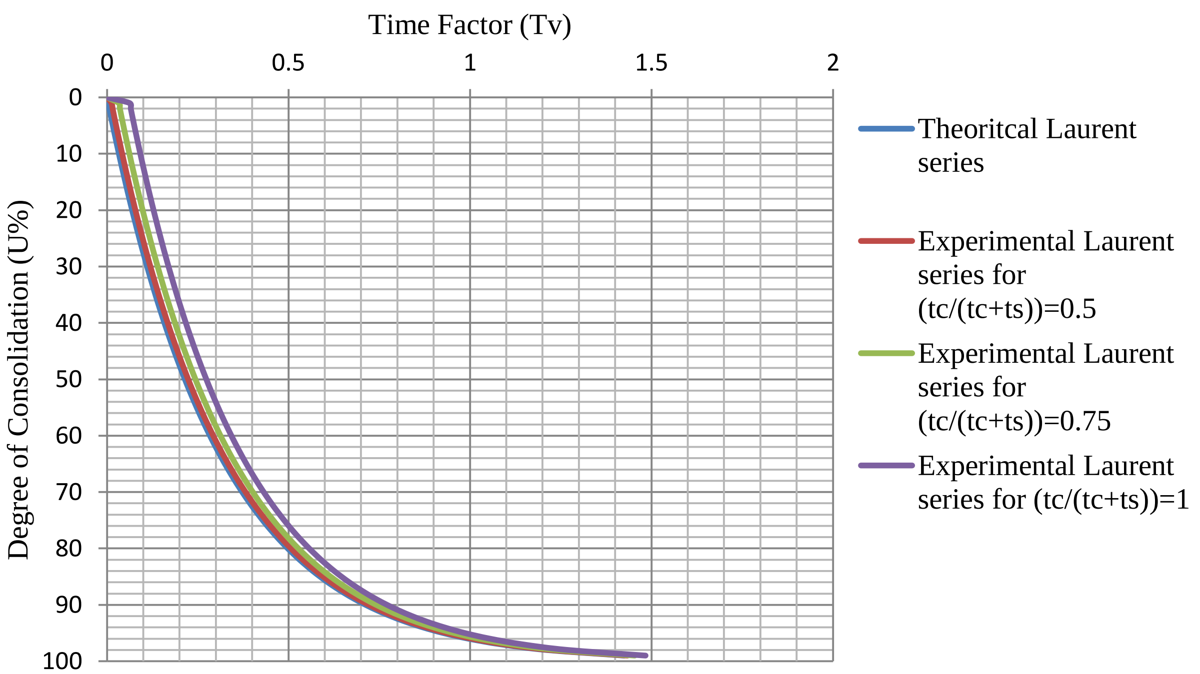

Fig 5.5: Variation of degree of consolidation with Laurent series.

From above observations of the dissipation of pore water pressure of silty sand it is clear that it converges with the results of the solution obtained from the solutions of the Laurent transform more than that of Fourier transform used by Terzaghi and hence validate this proposal. Above graphs provides clear vision of comparison for above said proposal.

5.4 Discussion

Thus from above analysis we can determine various parameters that decides the total degree of settelement in soil sample,containing both layers of sand as well as clay layer.Different tests were carried out and data comes to be as specific gravity of sample is 1.48 and from grain size distribution, soil sample resembles with that of fine sand having size more than 300 µm & for sand layer used C_u = 1.857 and C_c = 0.992 along with its specific gravity 2.85**.** Free swell index comes to be as 67.4% that means degree of expansion for these type of soil is generally very high.Consolidation behaviour is also well studied with the help of comparison of various theories mainly by Biot & Terzaghi**.**It is clear that in given soil strata, deformation of such porous media depends not only upon the stiffness of the porous material, but also upon the behaviour of the fluid in the pores.

If the permeability of the material is small, the deformations may be considerably hindered, or at least retarded, by the pore fluid. Thus in this way it is clear that resultant consolidation will not only takes place due to expulsion of pore water & release of stress but also due to restructuring of soil solids. The assumptions that the decrease in permeability is proportional to the decrease in compressibility during the consolidation process or permeability is proportional to the coefficient of consolidation holds good for the experiment being carried out.It is clear that being decreasing the thickness of clay layer & in place of it sand layer of corresponding thickness is used as a result of which ‘rate of consolidation’ increases and consolidation is achieved in faster rate. ‘Rate of consolidation’ increases at the rate of increase of thickness of sand layer.

CHAPTER – 6

CONCLUSION

1) Degree of consolidation by Laurent series and transformations is successfully analysed here and rate of consolidation obtained by Laurent transform is compared with that obtained from Fourier transform.

2) The Laurent transform is close to obtain rate of consolidation of soils of low permeability more clearly as compared to Fourier transform.

3) Since Laurent series is a type of convergent series therefore degree of consolidation for varied sample thickness and pore water pressure converges to a single value.

4) The nature of Laurent transform is transcendental. With the help of Laurent transform process of consolidation can be evaluated at microscopic level and for all kind of soils.

5) It is also helpful in determining consolidation of clays and can be applied to secondary consolidation superior to Fourier transform. It can be a source of very useful information like quick determination of consolidation characteristics, time compression data of the present, past and future, type and stage of consolidation, drainage conditions, load increment etc.

6) By using sand layer along with clay layer the stiffness of clay layer decreases and water from pores of clay layer get dissipated fast, hence void ratio decreases and soil layer get compacted there by achieving maximum degree of consolidation.

7) This analysis can be well supported and explained by transition of Fourier to Laurent transformation.

8) From the above analytical analysis of ‘degree of consolidation’ by two mathematical models i.e. by Fourier transformation & Laurent transformation,

appropriate predictions about completion of consolidation process for a given soil strata can be made.

9) It is clear that being decreasing the thickness of clay layer & in place of it sand layer of corresponding thickness is used as a result of which 'rate of consolidation' increases and consolidation is achieved in faster rate.

10) It is clear that being decreasing the thickness of clay layer & in place of it sand layer of corresponding thickness is used as a result of which 'rate of consolidation' increases and consolidation is achieved in faster rate.

11) Amount of dissipation of pore pressure can be determined at any instant during the process of consolidation.

12) Analytically from the curves of the two respective transformation it can be concluded that with the application of laurent transformation

    - Consolidation of stiff clays can be achieved in faster rate as compare to fourier series.
    - For fast decay of soils in terms of pore water pressure Laurent series will be more reliable.
    - It offers large boundaries domain that is from -∞ to +∞ and hence it is more comprehensive.

6.1 LIMITATION

1) Experimental data can not be perfectly corelated as per the two transforms used here,only near by conclusions are drawn till now.

2) Determination of imaginary(negative) roots for analysis of complete consolidation process is troublesome as the function varies with in the domain (-∞ to +∞) unlike in fourier series.

3) Laurent series gives good result for clayey soils of less thickness only.

4) No such conclusion can be drawn for three-dimensional consolidation using Laurent series

6.2 FUTURE SCOPE

1. Application of Laurent series for 3-Dimensional consolidation can be studied and analysed both analytically & experimentally.
2. Experimental validation of Laurent Series can be done for various types of soil.
3. For stiff clays, degree of consolidation at different time factor is carried out.
4. Comparison of rate of consolidation for different kinds of soil had to be done & verified by Laurent series.

CHAPTER-8

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APPENDIX

Uav	Tv (Fourier series with real roots)	Tv (Fourier series with imaginary roots)	Tv (Laurent series)
0	0	0	0
1	0.0025	0.0115	0.0215
2	0.005	0.014	0.024
3	0.0075	0.0165	0.0265
4	0.0101	0.0191	0.0291
5	0.0126	0.0216	0.0316
6	0.0153	0.0243	0.0343
7	0.0179	0.0269	0.0369
8	0.0206	0.0296	0.0396
9	0.0232	0.0322	0.0422
10	0.026	0.035	0.045
11	0.0287	0.0377	0.0477
12	0.0315	0.0405	0.0505
13	0.0343	0.0433	0.0533
14	0.0372	0.0462	0.0562
15	0.0401	0.0491	0.0591
16	0.043	0.052	0.062
17	0.0459	0.0549	0.0649
18	0.0489	0.0579	0.0679
19	0.0519	0.0609	0.0709
20	0.055	0.064	0.074
21	0.0581	0.0671	0.0771
22	0.0612	0.0702	0.0802
23	0.0644	0.0734	0.0834
24	0.0676	0.0766	0.0866
25	0.0709	0.0799	0.0899
26	0.0742	0.0832	0.0932
27	0.0776	0.0866	0.0966
28	0.081	0.09	0.1
29	0.0844	0.0934	0.1034
30	0.0879	0.0969	0.1069
31	0.0914	0.1004	0.1104
32	0.095	0.104	0.114
33	0.0987	0.1077	0.1177
34	0.1024	0.1114	0.1214
35	0.1062	0.1152	0.1252
36	0.11	0.119	0.129
37	0.1139	0.1229	0.1329

Uav	Tv (Fourier series with real roots)	Tv (Fourier series with imaginary roots)	Tv (Laurent series)
38	0.1178	0.1268	0.1368
39	0.1218	0.1308	0.1408
40	0.1259	0.1359	0.1459
41	0.13	0.14	0.15
42	0.1342	0.1442	0.1542
43	0.1385	0.1485	0.1585
44	0.1429	0.1529	0.1629
45	0.1473	0.1573	0.1673
46	0.1518	0.1618	0.1718
47	0.1564	0.1664	0.1764
48	0.1611	0.1711	0.1811
49	0.1659	0.1759	0.1859
50	0.1708	0.1808	0.1908
51	0.1758	0.1858	0.1958
52	0.1809	0.1909	0.2009
53	0.186	0.196	0.206
54	0.1913	0.2013	0.2113
55	0.1968	0.2068	0.2168
56	0.2023	0.2123	0.2223
57	0.208	0.218	0.228
58	0.2138	0.2238	0.2338
59	0.2197	0.2297	0.2397
60	0.2258	0.2358	0.2458
61	0.232	0.242	0.252
62	0.2384	0.2484	0.2584
63	0.245	0.255	0.265
64	0.2517	0.2617	0.2717
65	0.2587	0.2687	0.2787
66	0.2658	0.2758	0.2858
67	0.2732	0.2832	0.2932
68	0.2808	0.2908	0.3008
69	0.2886	0.2986	0.3086
70	0.2967	0.3067	0.3167
71	0.305	0.315	0.325
72	0.3134	0.3234	0.3334
73	0.3226	0.3326	0.3426
74	0.3319	0.3419	0.3519
75	0.3416	0.3516	0.3616
76	0.3517	0.3617	0.3717
77	0.3621	0.3721	0.3821
78	0.3731	0.3831	0.3931

Uav	Tv (Fourier series with real roots)	Tv (Fourier series with imaginary roots)	Tv (Laurent series)
79	0.3846	0.3946	0.4046
80	0.3966	0.4066	0.4166
81	0.409	0.419	0.429
82	0.4225	0.4325	0.4425
83	0.4366	0.4466	0.4566
84	0.4516	0.4616	0.4716
85	0.4675	0.4775	0.4875
86	0.4845	0.4945	0.5045
87	0.5027	0.5127	0.5227
88	0.5225	0.5325	0.5425
89	0.5439	0.5539	0.5639
90	0.5674	0.5774	0.5874
91	0.5933	0.6033	0.6133
92	0.6224	0.6324	0.6424
93	0.6553	0.6653	0.6753
94	0.6932	0.7032	0.7132
95	0.7382	0.7482	0.7582
96	0.7932	0.8032	0.8132
97	0.864	0.874	0.884
98	0.964	0.974	0.984
99	1.1347	1.1447	1.1547